Informath:

Interlingual Informalization and Autoformalization with Dedukti and GF

LORIA Nancy 24 July 2025

(v4 FAU Erlangen 18 July 2025,
v3 CIIRC, TU Prague 14 July 2025,
v2 Chalmers/GU 25 April 2025,
v1 LMF/ENS Saclay 10 April 2025)

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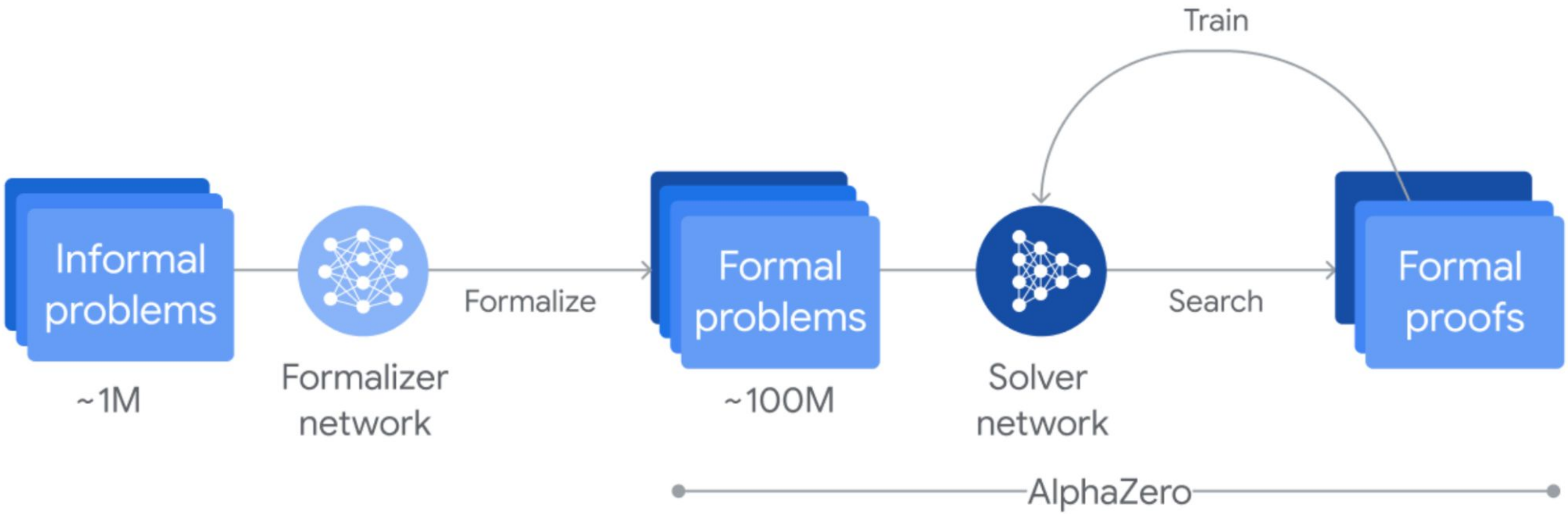
Prologue:

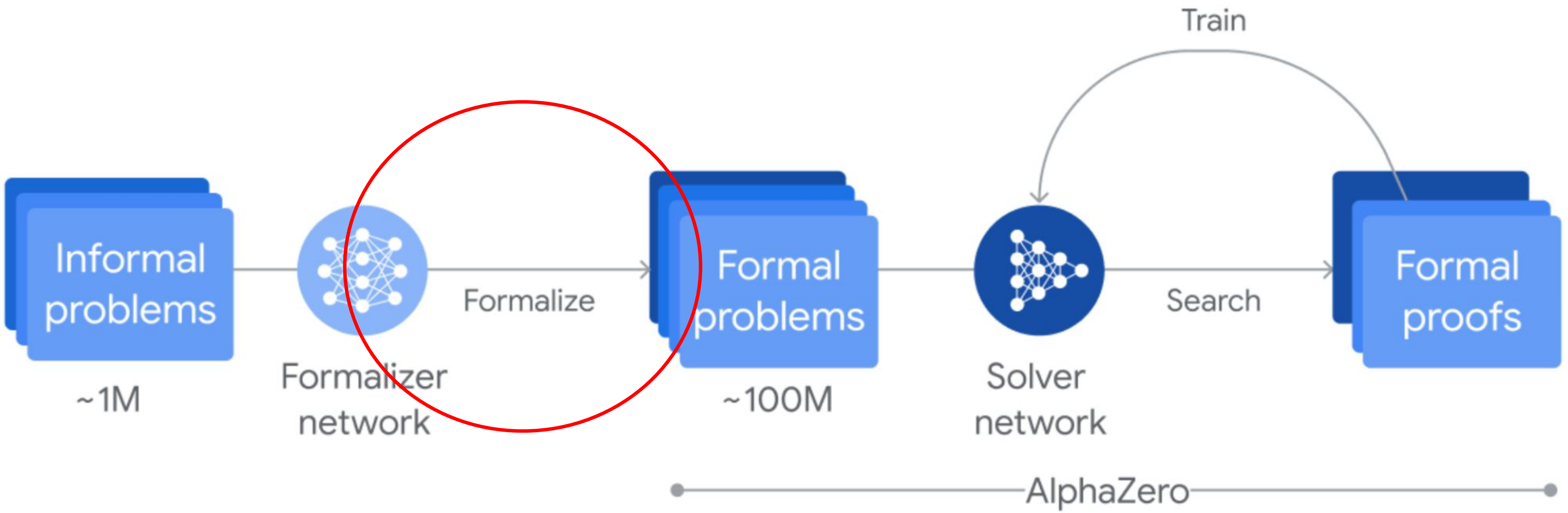
AlphaProof and Autoformalization

AI achieves silver-medal standard solving International Mathematical Olympiad problems



https://deepmind.google/discover/blog/ai-solves-imo-problems-at-silver-medal-level/

AI achieves silver-medal standard solving International Mathematical Olympiad problems



"First, the problems were manually translated into formal mathematical language for our systems to understand."

https://deepmind.google/discover/blog/ai-solves-imo-problems-at-silver-medal-level/

(In)formalization, symbolic CNL

Trybulec 1973: Mizar

Coscoy, Kahn & Théry 1994: Coq proofs to text

Wenzel 1999: Isabelle-Isar

Hallgren & Ranta 2000: GF-Alfa (Agda)

Paskevich 2007: ForTheL

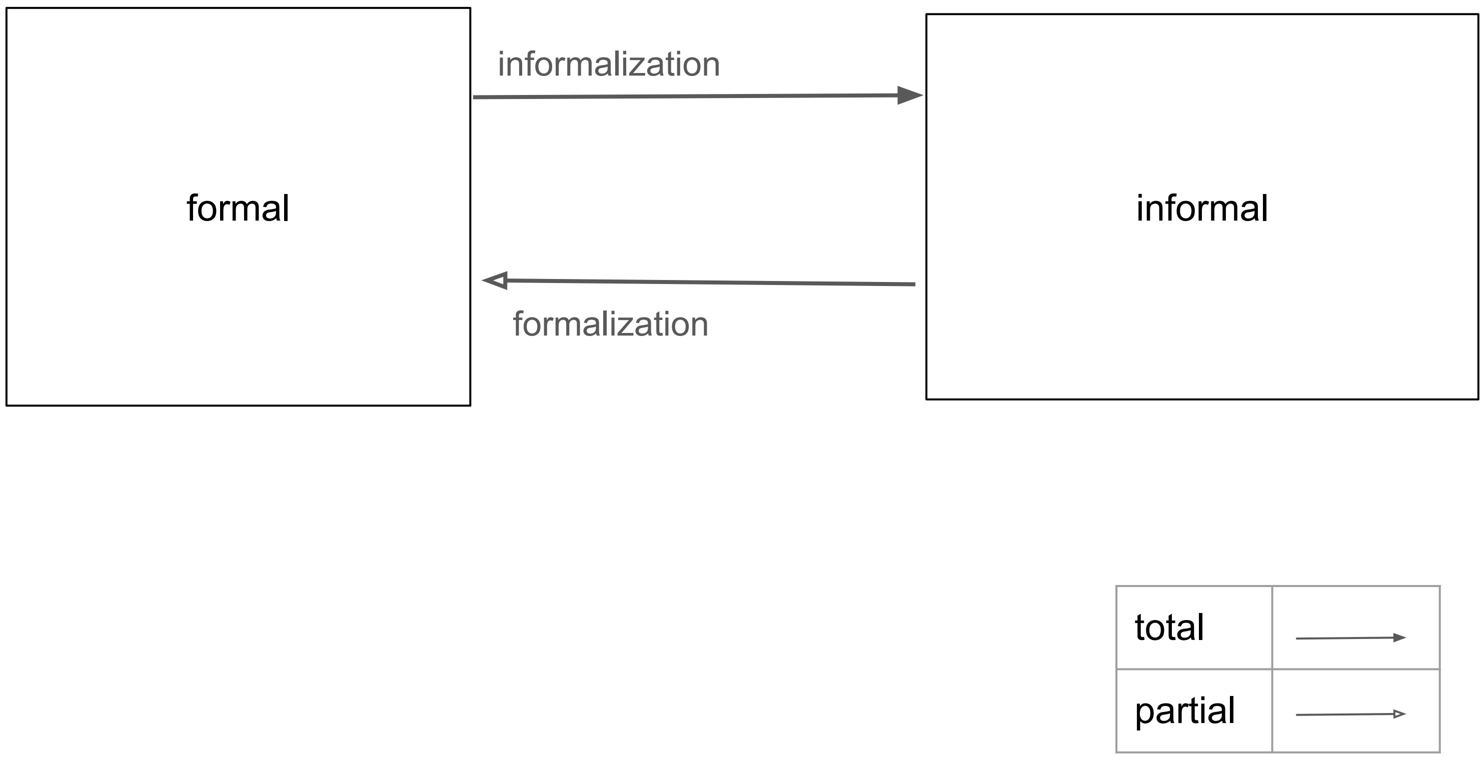
Cramer, Koepke & al 2009: Naproche

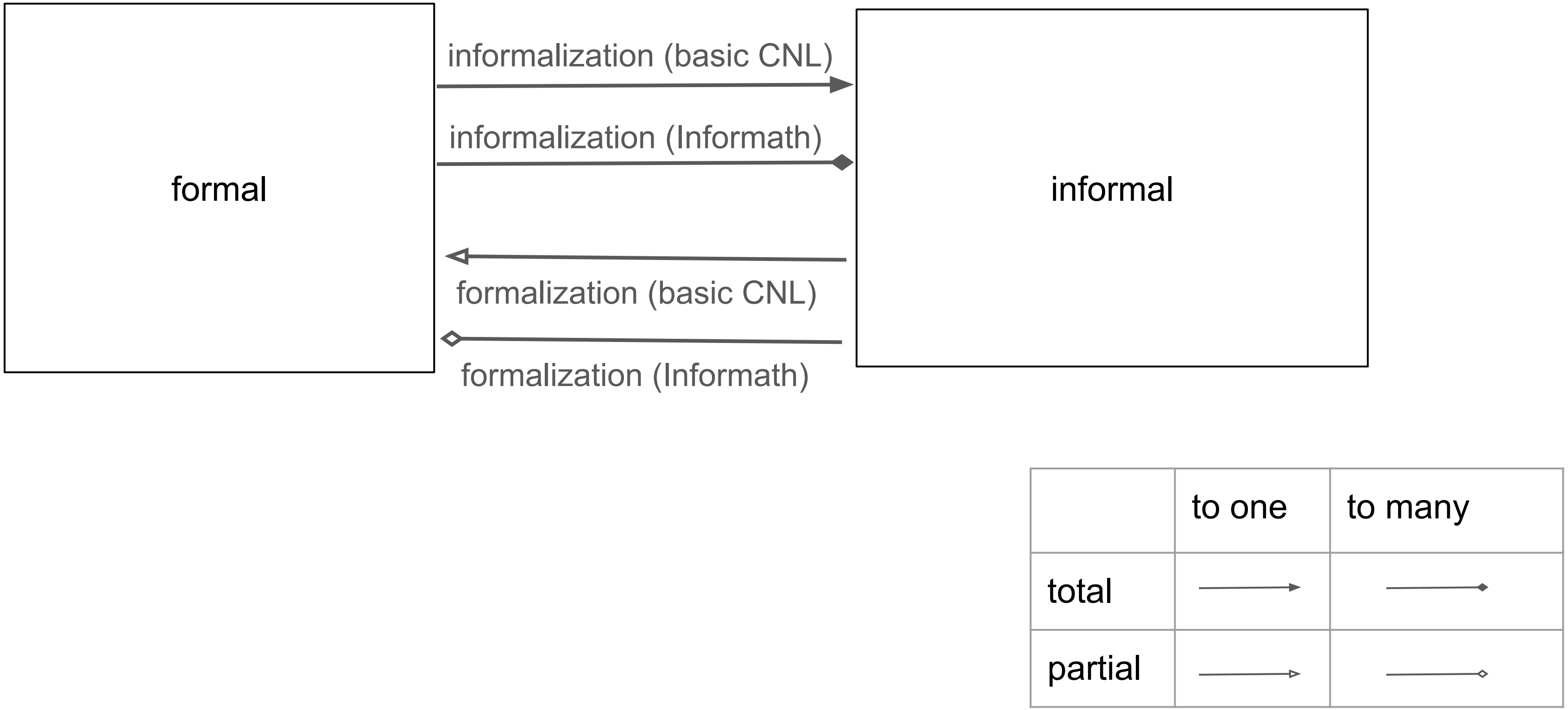
Humayoun & Raffalli 2011: MathNat

Pathak 2023: GF-Lean

Massot 2024: Verbose-Lean4

Kelber, Kohlhase, Schaefer & Schütz: Flexiformal mathematics, 2025

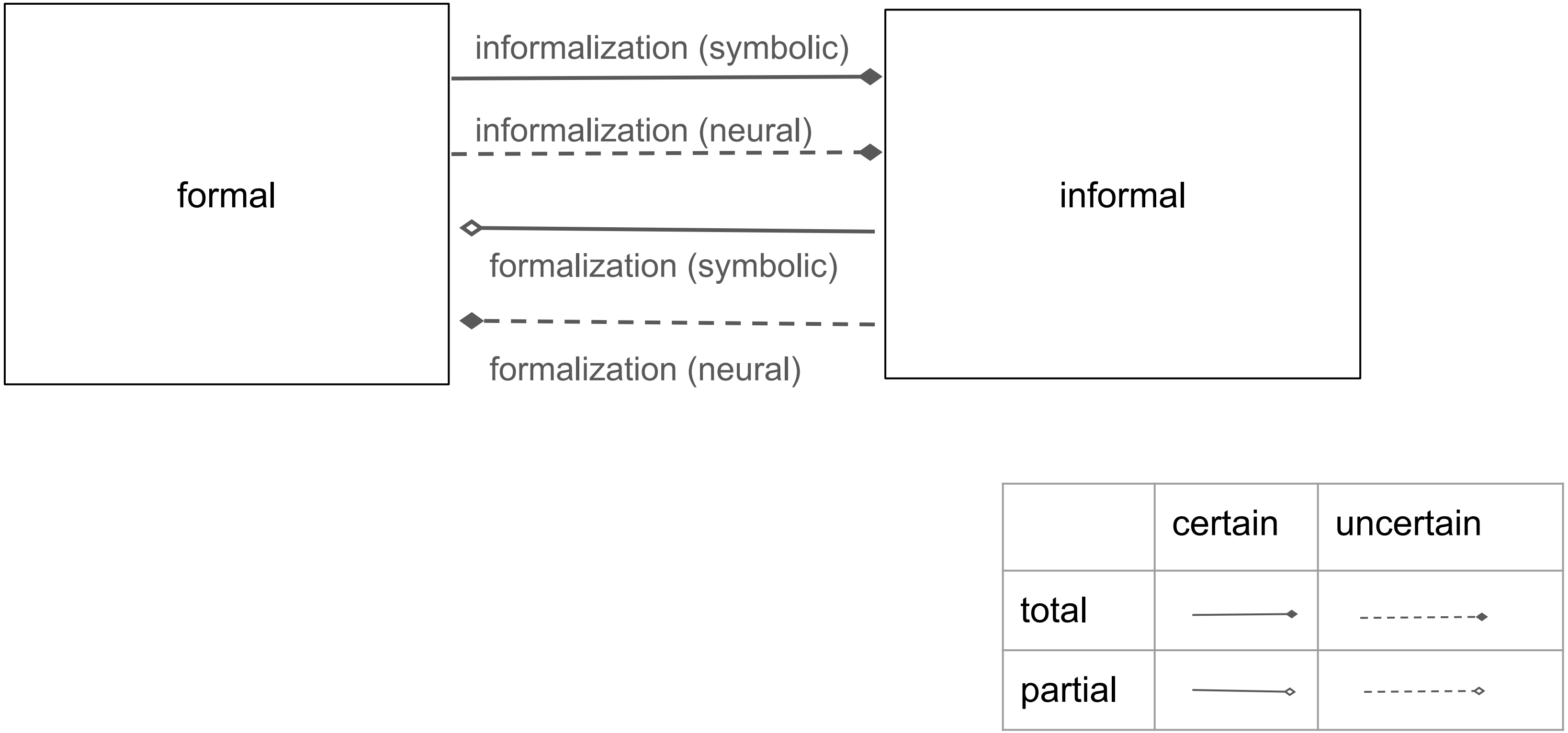

informalization
formal
informal
formalization
total
partial


formal
informal
informalization (basic CNL)
informalization (Informath)
formalization (basic CNL)
formalization (Informath)
to one
to many
total
partial

Autoformalization, neural

Wang, Kaliczyk & Urban 2018: NMT and Mizar

Wu, Jiang, Li, Rabe, Staats, Jamnik & Szegedy 2022 : autoformalization with LLM


formal
informal
informalization (symbolic)
informalization (neural)
formalization (symbolic)
formalization (neural)
certain
uncertain
total
partial

Don't guess if you know.

- there is no essential need for neural informalization
- (except its allegedly low cost)

- However, (auto)formalization may require guessing

- symbolic informalization has things to contribute even here
    - synthetic data generation
    - verification feedback

Multi-language Diversity Benefits Autoformalization

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- "informalisation is much easier than formalisation"

- uses an GPT-4 to produce the dataset MMA to fine-tune LLaMA
    - ~70% "more or less acceptable"

- resulting autoformalization:
    - 16-18% "acceptable with minimal corrections"

- ~~symbolic informalization~~

NeurIPS 2024,

"symbolic informalisation tools

- result in natural language content that lacks the inherent diversity and flexibility in expression: they are rigid and not natural-language-like.
- symbolic informalisation tools are hard to design and implement
- They also differ a lot for different formal languages, hence the approach is not scalable for multiple formal languages. "

https://arxiv.org/abs/2311.03755

Informath

The goal of Informath

Symbolic informalization that

- results in natural language content that *has* ~~lacks~~ the inherent diversity and flexibility in expression: they are ~~rigid and not~~ natural-language-like.
- symbolic informalisation tools are *feasible* ~~hard~~ to design and implement ***with proper methods***
- They *can be shared* ~~also differ a lo~~t for different formal languages, hence the approach is ~~not~~ scalable for multiple formal languages. ***And even for multiple natural languages.***

Agda:

```
postulate prop110 :
  (a : Int) -> (c : Int) ->
  and (odd a) (odd c) -> all Int (\ b ->
    even (plus (times a b) (times b c)))
```

Prop110. Let $a, c \in Z$. Assume that both a and c are odd. Then $ab + bc$ is even for all integers b.

Rocq:

```
prop110 : forall a : Int, forall c : Int,
  (odd a /\ odd c -> forall b : Int,
   even (a * b + b * c)) .
```

Prop110. Soient $a, c \in Z$. Supposons que a et c sont impairs. Alors $ab + bc$ est pair pour tous les entiers b.

Lean:

```
prop110 (a c : Int) (x : odd a ∧ odd c)
:
  ∀ b : Int, even (a * b + b * c)
```

Prop110. Låt $a, c \in Z$. Anta att både a och c är udda. Då är $ab + bc$ jämnt för alla heltal b.

Agda:

```
postulate prop110 :
  (a : Int) -> (c : Int) ->
  and (odd a) (odd c) -> all Int (\ b ->
    even (plus (times a b) (times b c)))
```

Rocq:

```
prop110 : forall a : Int, for
  (odd a /\ odd c -> forall b
   even (a * b + b * c)) .
```

Dedukti:

```
prop110 : (a : Elem Int) ->
  (c : Elem Int) ->
    Proof (and (odd a)
   (odd c)) ->
    Proof (forall Int
   (b => even (plus
   (times a b) (times b c)))).
```

Lean:

```
prop110 (a c : Int) (x : odd a ∧ odd c)
:
  ∀ b : Int, even (a * b + b * c)
```

Prop110. Let $a, c \in Z$. Assume that both a and c are odd. Then $ab + bc$ is even for all integers b.

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Rocq:

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prop110 : (a : Elem Int) ->
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    Proof (and (odd a)
   (odd c)) ->
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```

Lean:

```
prop110 (a c : Int) (x : odd a ∧ odd c)
:
  ∀ b : Int, even (a * b + b * c)
```

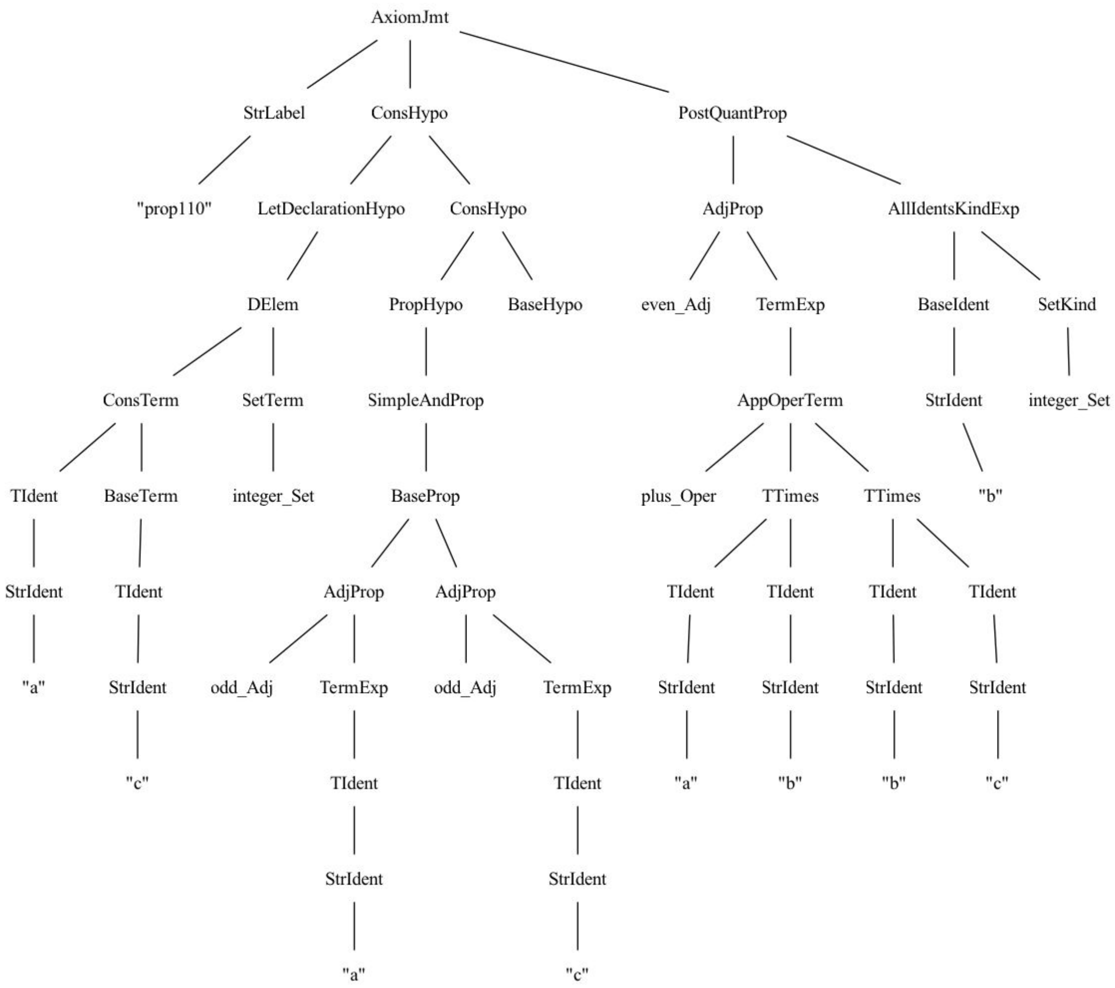
Prop110. Let $a, c \in Z$. Assume that both a and c are odd. Then $ab + bc$ is even for all integers b.

GF:

```
AxiomJmt (StrLabel "prop110")
(ConsHypo (LetFormulaHypo (FElem
(ConsTerm (TIdent (StrIdent "a"))
(BaseTerm (TIdent (StrIdent "c"))))
(SetTerm integer_Set))) (ConsHypo
(PropHypo (AdjProp odd_Adj (AndExp
(BaseExp (TermExp (TIdent (StrIdent
"a"))) (TermExp (TIdent (StrIdent
"c"))))))) BaseHypo)) (PostQuantProp
(AdjProp even_Adj (TermExp
(AppOperTerm plus_Oper (TTimes (TIdent
(StrIdent "a")) (TIdent (StrIdent
"b"))) (TTimes (TIdent (StrIdent "b"))
(TIdent (StrIdent "c")))))))
(AllIdentsKindExp (BaseIdent (StrIdent
"b")) (SetKind integer_Set)))
```

…t $a, c \in Z$. Supposons que …airs. Alors $ab + bc$ est pair …ntiers b.

Prop110. Låt $a, c \in Z$. Anta att både a och c är udda. Då är $ab + bc$ jämnt för alla heltal b.


AxiomJmt
StrLabel
ConsHypo
PostQuantProp
"prop110"
LetDeclarationHypo
ConsHypo
AdjProp
AllIdentsKindExp
DElem
PropHypo
BaseHypo
even_Adj
TermExp
BaseIdent
SetKind
ConsTerm
SetTerm
SimpleAndProp
AppOperTerm
StrIdent
integer_Set
TIdent
BaseTerm
integer_Set
BaseProp
plus_Oper
TTimes
TTimes
"b"
StrIdent
TIdent
AdjProp
AdjProp
TIdent
TIdent
TIdent
TIdent
"a"
StrIdent
odd_Adj
TermExp
odd_Adj
TermExp
StrIdent
StrIdent
StrIdent
StrIdent
"c"
TIdent
TIdent
"a"
"b"
"b"
"c"
StrIdent
StrIdent
"a"
"c"

Interlude: GF

GF = Grammatical Framework

GF = Logical Framework + Grammar

First release 1998 at Xerox Research Centre Europe, Grenoble

Based on earlier work with ALF (Another LF, predecessor of Agda) 1992

https://www.grammaticalframework.org/

Abstract and concrete syntax: judgements

```
-- abstract syntax = LF

cat C  Γ

fun f : T

def t = u
```

```
-- concrete syntax

lincat C = L

lin f = t

param P = C | … | C

oper h : T = t
```

Abstract and concrete syntax: examples

```
-- abstract syntax = LF

cat Prop ; Term

fun commutative : Term -> Prop

def commutative f =
  forall Obj (\x, y ->
     Id Obj (f x y) (f y x)
```

```
-- concrete syntax

lincat Prop, Term = Str

lin commutative x =
     x ++ "is commutative"
```

Concrete syntax: parameters and operations

```
-- abstract syntax = LF

cat Prop ; Term

fun commutative : Term -> Prop
```

```
-- concrete syntax for English

lincat
  Prop = Str
  Term = {s : Str ; n : Number}

lin commutative x = x.s ++
   copula ! x.n ++ "commutative"

param
  Number = Sg | Pl

oper
  copula : Number => Str =
    table {Sg => "is" ; Pl => "are"}
```

Concrete syntax: parameters and operations

```
-- abstract syntax = LF

cat Prop ; Term

fun commutative : Term -> Prop
```

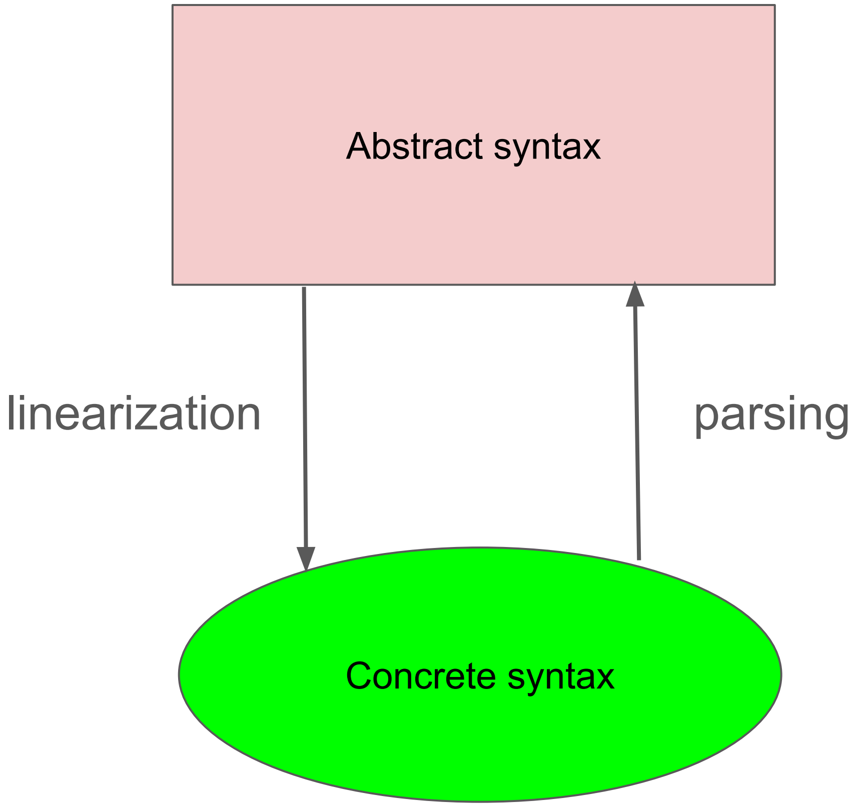
```
-- concrete syntax for French

lincat
  Prop = Mood => Str
  Term = {s : Str ; g : Gender ; n : Number}

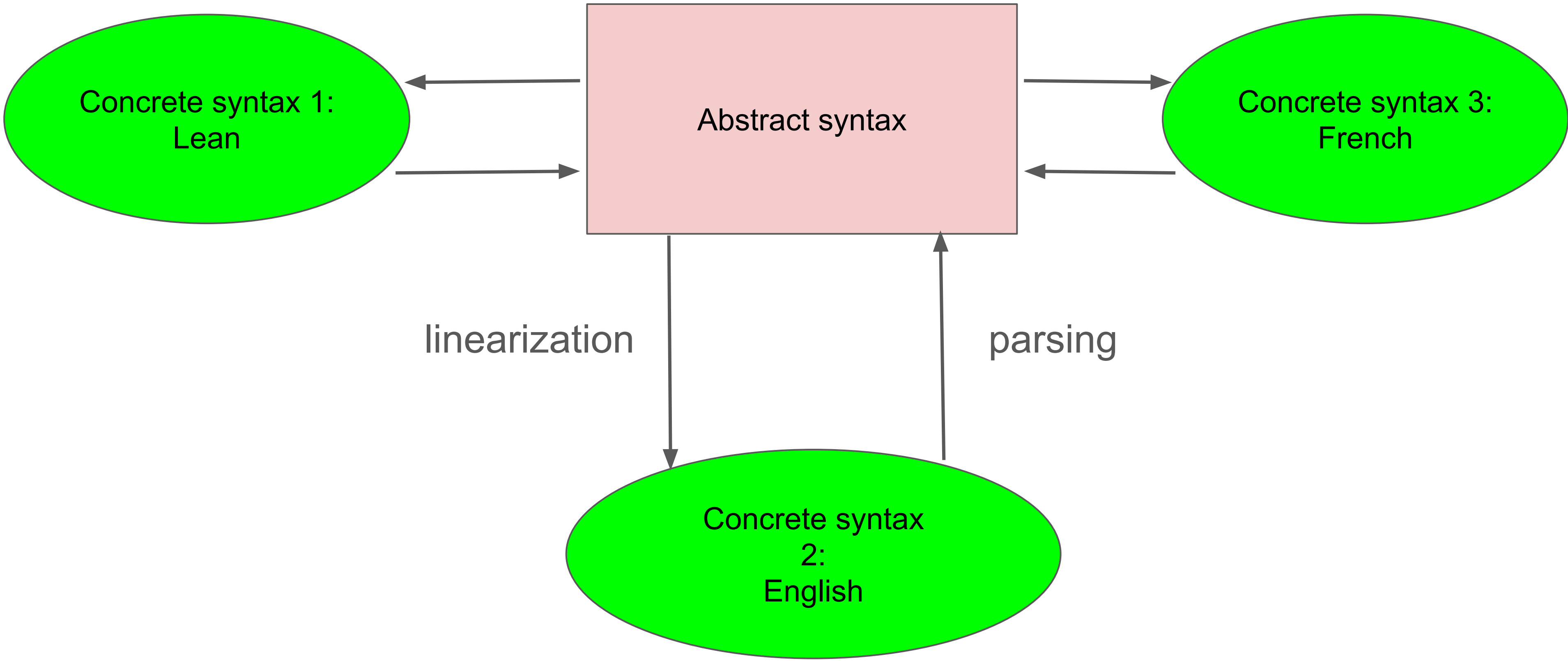
lin commutative x = \\m => x.s ++
   copula ! m ! n ++
   mkA "commutatif" ! x.g ! x.n

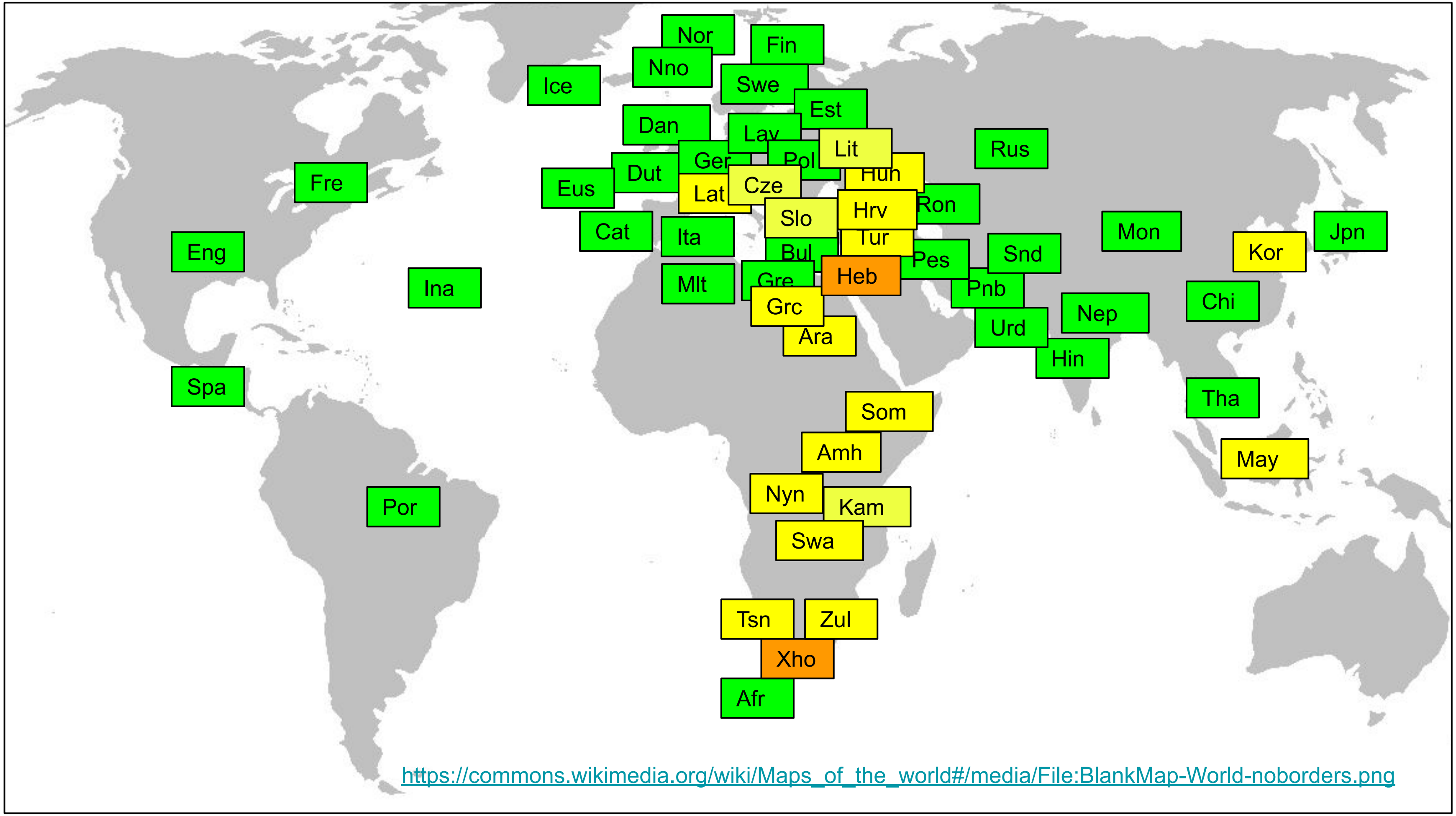
param
   Number = Sg | Pl
   Gender = Masc | Fem
   Mood = Ind | Subj
oper
  mkA : Str -> Gender => Number = Str = ...
  copula : Mood => Number => Str = ...
```

Reversible mappings



Multilingual grammars





RGL = Resource Grammars Library, created by the GF community 2001-2025

RGL = Resource Grammar Library

morphology and syntax for ~50 languages

```
-- inflection of French adjectives, slightly simplified

mkA : Str -> A = \adj ->
    case adj of {
      _ + "eux"=> <adj, init adj + "se", adj, init adj + "ses"> ;
      _ + "al" => <adj, adj + "e", init adj + "ux", adj + "es"> ;
      _ + "en" => <adj, adj + "ne", adj + "s", adj + "nes"> ;
      _ + "el" => <adj, adj + "le", adj + "s", adj + "les"> ;
      x + "er" => <adj, x + "ère", adj + "s", x + "ères"> ;
      _ + "if" => <adj, init adj + "ve", adj + "s", init adj + "ves"> ;
      _ + "s"  => <adj, adj + "e", adj, adj + "es"> ;
      _ + "e"  => <adj, adj, adj + "s", adj + "s"> ;
      _        => <adj, adj + "e", adj + "s", adj + "es">
      } ;
```

http://www.grammaticalframework.org/lib/doc/synopsis/

RGL

syntactic combination API

shared by all languages in the library

usable as functor interface + instances

http://www.grammaticalframework.org/lib/doc/synopsis/

Concrete syntax: functor over the RGL

```
-- abstract syntax code

cat Prop ; Term
fun commutative : Term -> Prop


-- shared functor code

lincat
  Prop = Cl
  Term = NP

lin
  commutative x =
     mkCl x commutative_A
```

```
-- added code for each language

-- Eng
  commutative_A =
    mkA "commutative"

-- Fre
  commutative_A =
    mkA "commutatif"

-- Fin
  commutative_A =
    mkA "kommutatiivinen"
```

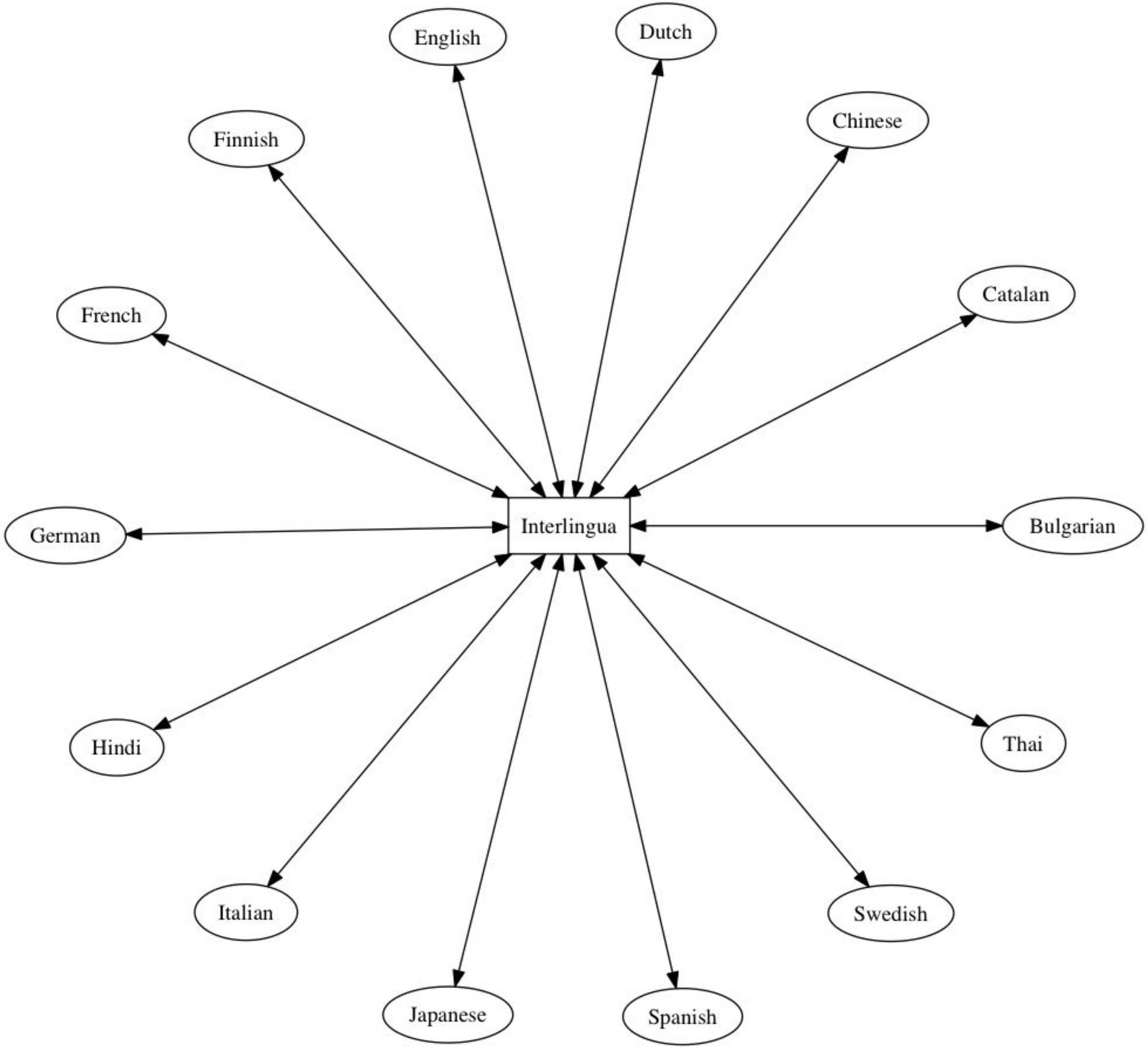
Context-free expansions of `'commutative : Term -> Prop'`

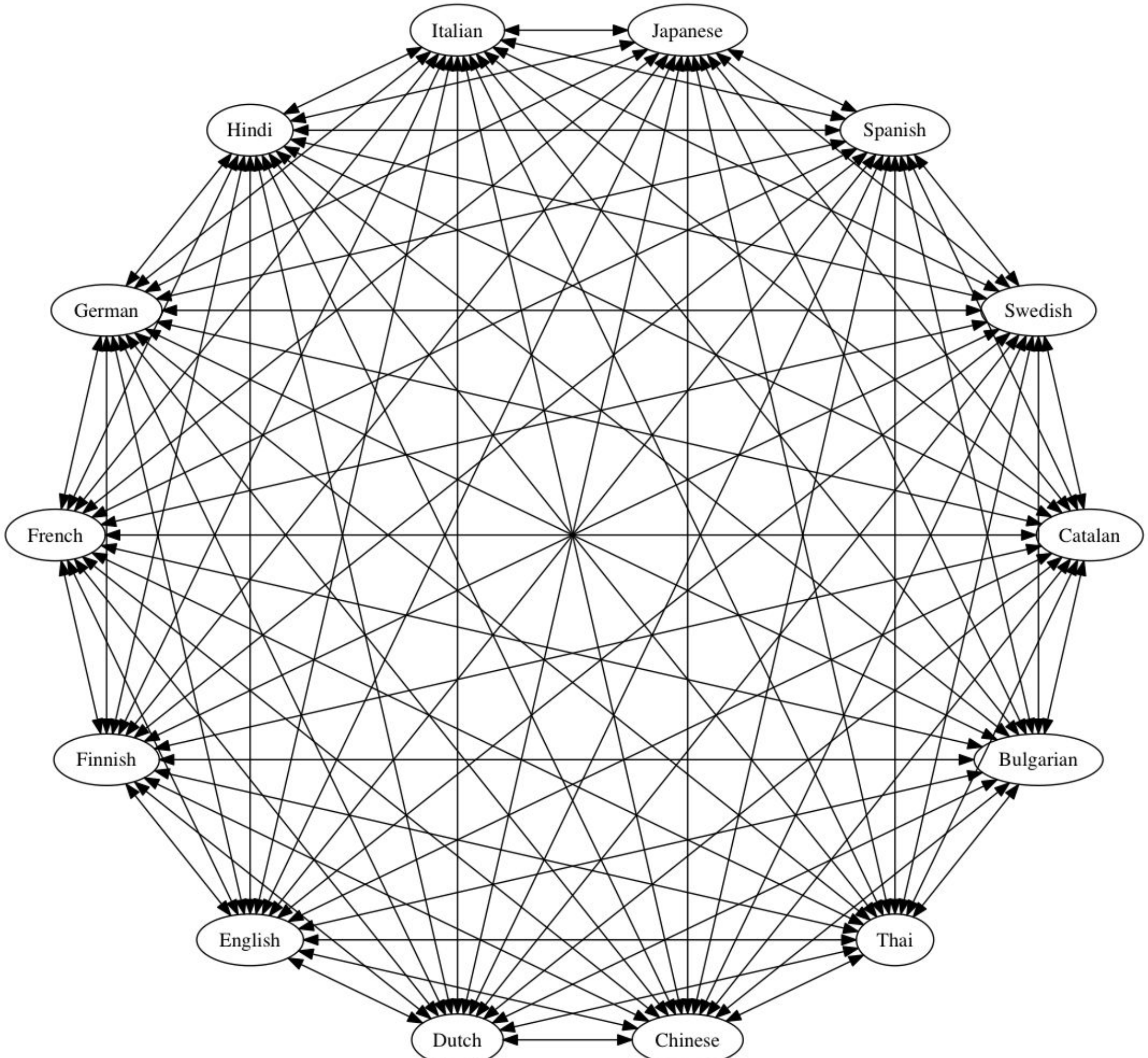
```
Prop_1_0  ::= Term_5 "is" "commutative"
Prop_1_0  ::= Term_6 "are" "commutative"
Prop_1_2  ::= "are" Term_6 "commutative"
Prop_1_2  ::= "is" Term_5 "commutative"
Prop_1_3  ::= Term_5 "is" "not" "commutative"
Prop_1_3  ::= Term_6 "are" "not" "commutative"
Prop_1_5  ::= "are" Term_6 "not" "commutative"
Prop_1_5  ::= "is" Term_5 "not" "commutative"
Prop_1_6  ::= Term_5 "isn't" "commutative"
Prop_1_6  ::= Term_6 "aren't" "commutative"
Prop_1_7  ::= Term_5 "isn't" "commutative"
Prop_1_7  ::= Term_6 "aren't" "commutative"
Prop_1_8  ::= "aren't" Term_6 "commutative"
Prop_1_8  ::= "isn't" Term_5 "commutative"
```

Context-free expansions of 'commutative : Term -> Prop'

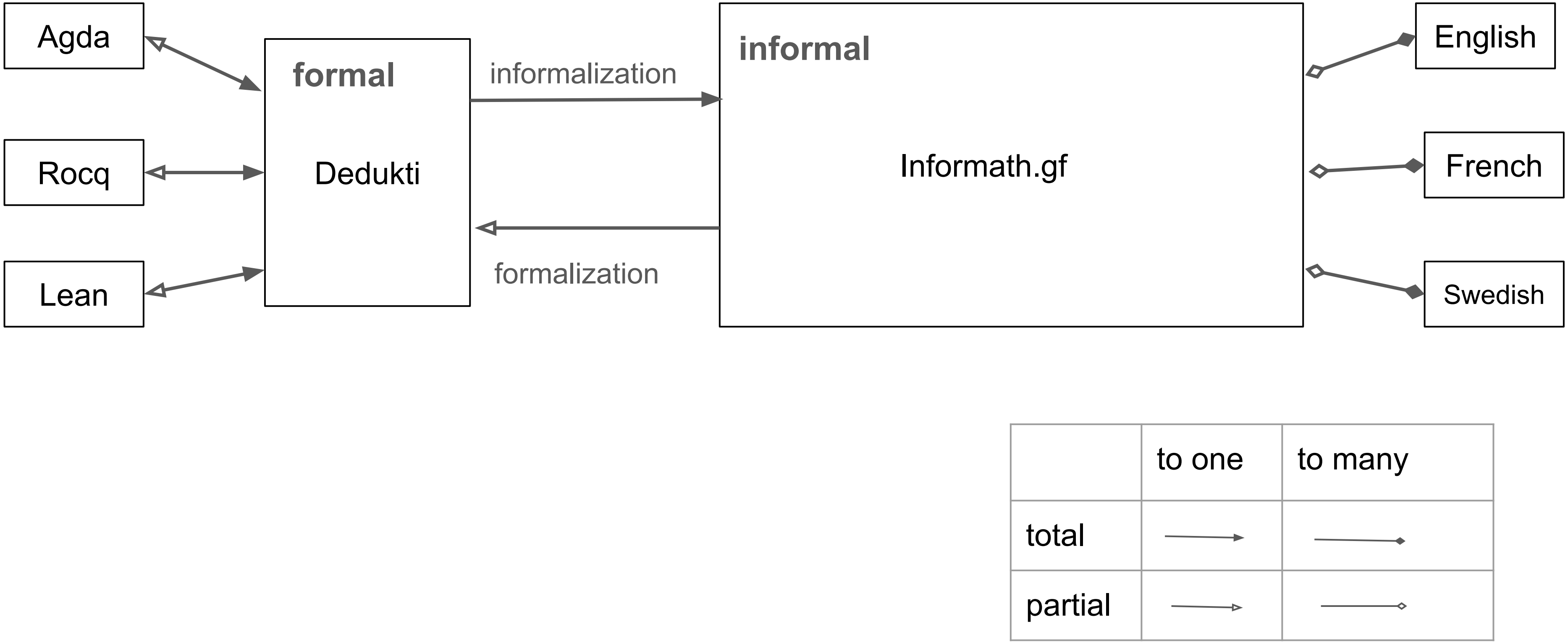
```
Prop_1_0  ::= Term_5 "is" "commutative"
Prop_1_0  ::= Term_6 "are" "commutative"
Prop_1_2  ::= "are" Term_6 "commutative"
Prop_1_2  ::= "is" Term_5 "commutative"
Prop_1_3  ::= Term_5 "is" "not" "commutative"
Prop_1_3  ::= Term_6 "are" "not" "commutative"
Prop_1_5  ::= "are" Term_6 "not" "commutative"
Prop_1_5  ::= "is" Term_5 "not" "commutative"
Prop_1_6  ::= Term_5 "isn't" "commutative"
Prop_1_6  ::= Term_6 "aren't" "commutative"
Prop_1_7  ::= Term_5 "isn't" "commutative"
Prop_1_7  ::= Term_6 "aren't" "commutative"
Prop_1_8  ::= "aren't" Term_6 "commutative"
Prop_1_8  ::= "isn't" Term_5 "commutative"
```

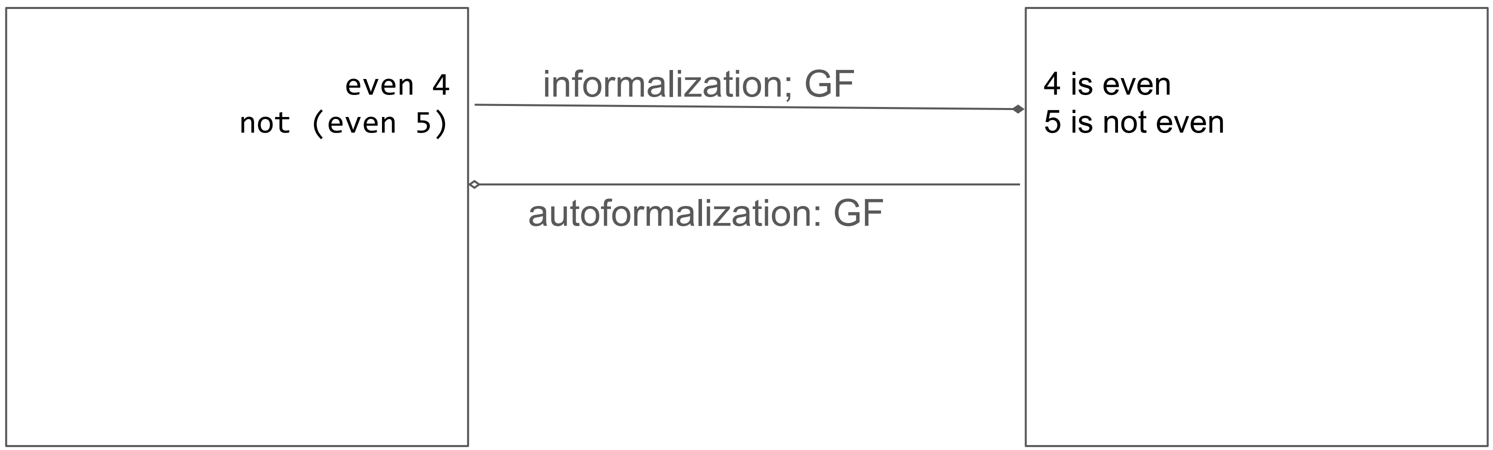
```
Prop_1_0  ::= Term_1 "est" "commutatif"
Prop_1_0  ::= Term_2 "n'est" "commutatif"
Prop_1_0  ::= Term_3 "sont" "commutatifs"
Prop_1_0  ::= Term_4 "ne" "sont" "commutatifs"
Prop_1_1  ::= Term_1 "soit" "commutatif"
Prop_1_1  ::= Term_2 "ne" "soit" "commutatif"
Prop_1_1  ::= Term_3 "soient" "commutatifs"
Prop_1_1  ::= Term_4 "ne" "soient" "commutatifs"
Prop_1_10 ::= "n'est" Term_1 "commutatif"
Prop_1_10 ::= "n'est" Term_2 "commutatif"
Prop_1_10 ::= "ne" "sont" Term_3 "commutatifs"
Prop_1_10 ::= "ne" "sont" Term_4 "commutatifs"
Prop_1_11 ::= "ne" "soient" Term_3 "commutatifs"
Prop_1_11 ::= "ne" "soient" Term_4 "commutatifs"
Prop_1_11 ::= "ne" "soit" Term_1 "commutatif"
Prop_1_11 ::= "ne" "soit" Term_2 "commutatif"
Prop_1_2  ::= Term_1 "n'est" "pas" "commutatif"
Prop_1_2  ::= Term_2 "n'est" "pas" "commutatif"
Prop_1_2  ::= Term_3 "ne" "sont" "pas" "commutatifs"
Prop_1_2  ::= Term_4 "ne" "sont" "pas" "commutatifs"
Prop_1_3  ::= Term_1 "ne" "soit" "pas" "commutatif"
Prop_1_3  ::= Term_2 "ne" "soit" "pas" "commutatif"
Prop_1_3  ::= Term_3 "ne" "soient" "pas" "commutatifs"
Prop_1_3  ::= Term_4 "ne" "soient" "pas" "commutatifs"
Prop_1_4  ::= Term_1 "n'est" "commutatif"
Prop_1_4  ::= Term_2 "n'est" "commutatif"
Prop_1_4  ::= Term_3 "ne" "sont" "commutatifs"
Prop_1_4  ::= Term_4 "ne" "sont" "commutatifs"
Prop_1_5  ::= Term_1 "ne" "soit" "commutatif"
Prop_1_5  ::= Term_2 "ne" "soit" "commutatif"
Prop_1_5  ::= Term_3 "ne" "soient" "commutatifs"
Prop_1_5  ::= Term_4 "ne" "soient" "commutatifs"
Prop_1_6  ::= "est" Term_1 "commutatif"
Prop_1_6  ::= "n'est" Term_2 "commutatif"
Prop_1_6  ::= "ne" "sont" Term_4 "commutatifs"
Prop_1_6  ::= "sont" Term_3 "commutatifs"
Prop_1_7  ::= "ne" "soient" Term_4 "commutatifs"
Prop_1_7  ::= "ne" "soit" Term_2 "commutatif"
Prop_1_7  ::= "soient" Term_3 "commutatifs"
Prop_1_7  ::= "soit" Term_1 "commutatif"
Prop_1_8  ::= "n'est" "pas" Term_1 "commutatif"
Prop_1_8  ::= "n'est" "pas" Term_2 "commutatif"
Prop_1_8  ::= "ne" "sont" "pas" Term_3 "commutatifs"
Prop_1_8  ::= "ne" "sont" "pas" Term_4 "commutatifs"
Prop_1_9  ::= "ne" "soient" "pas" Term_3 "commutatifs"
Prop_1_9  ::= "ne" "soient" "pas" Term_4 "commutatifs"
Prop_1_9  ::= "ne" "soit" "pas" Term_1 "commutatif"
Prop_1_9  ::= "ne" "soit" "pas" Term_2 "commutatif"
```

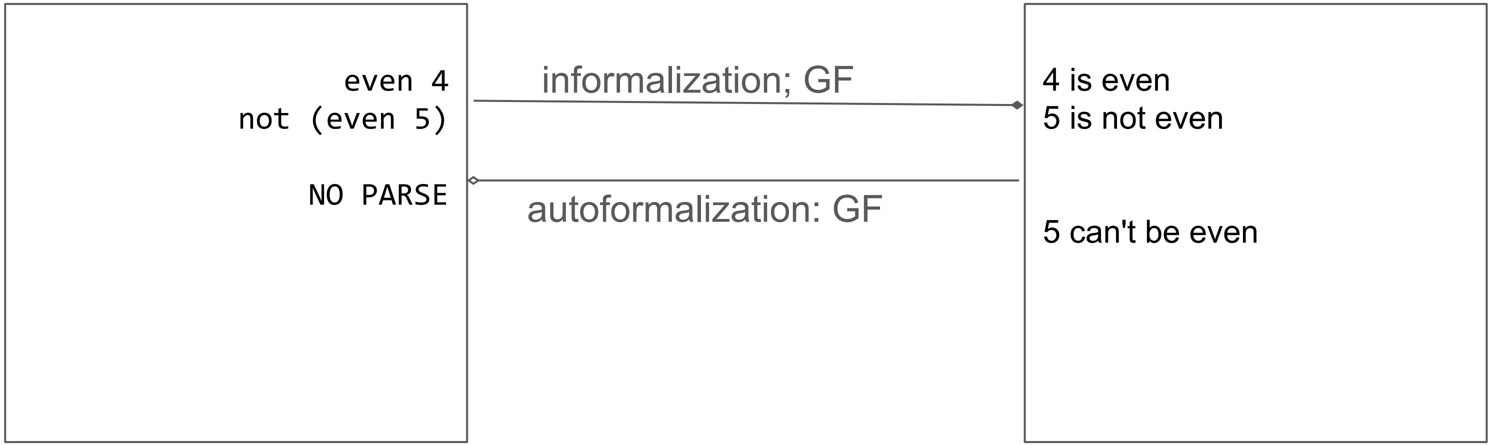

English
Dutch
Finnish
Chinese
French
Catalan
German
Interlingua
Bulgarian
Hindi
Thai
Italian
Swedish
Japanese
Spanish

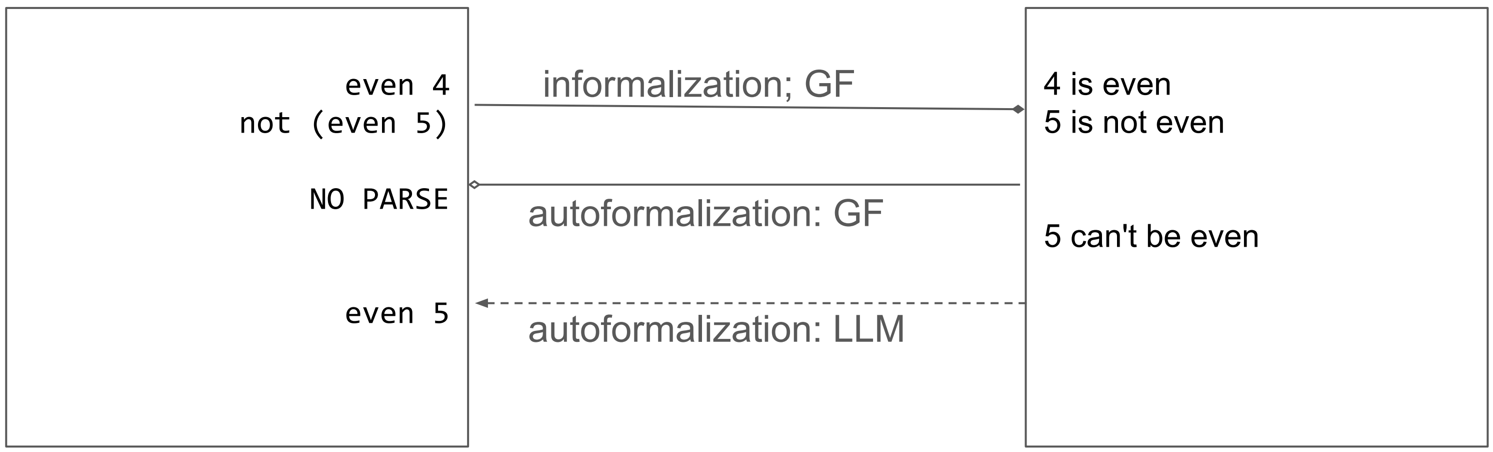

Italian
Japanese
Hindi
Spanish
German
Swedish
French
Catalan
Finnish
Bulgarian
English
Thai
Dutch
Chinese

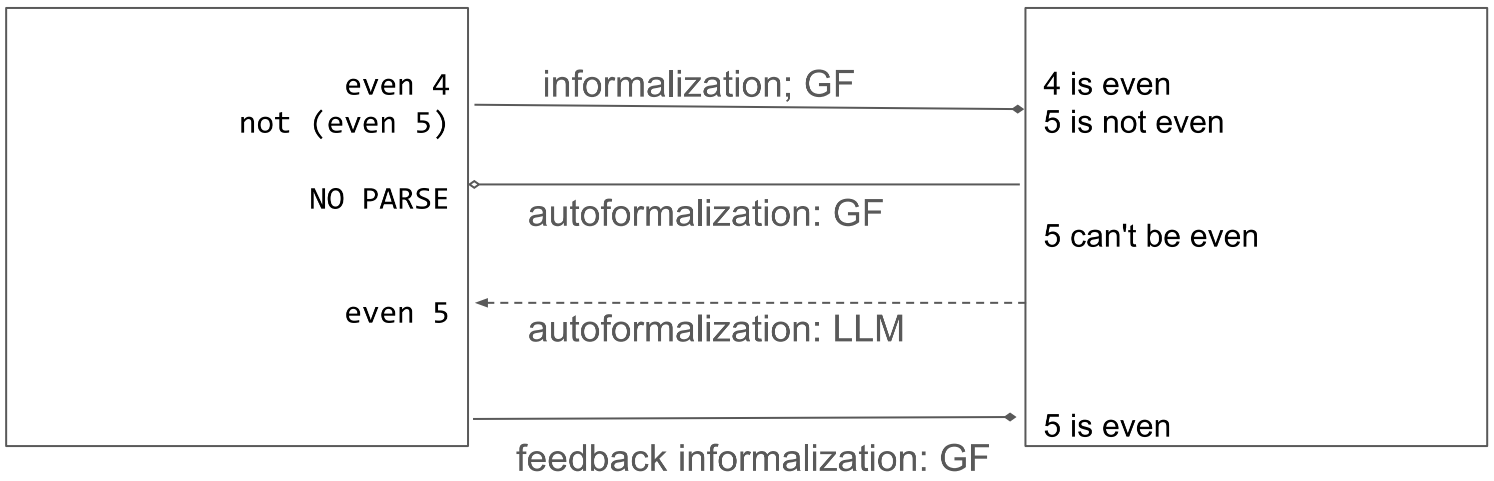
Back to Informath

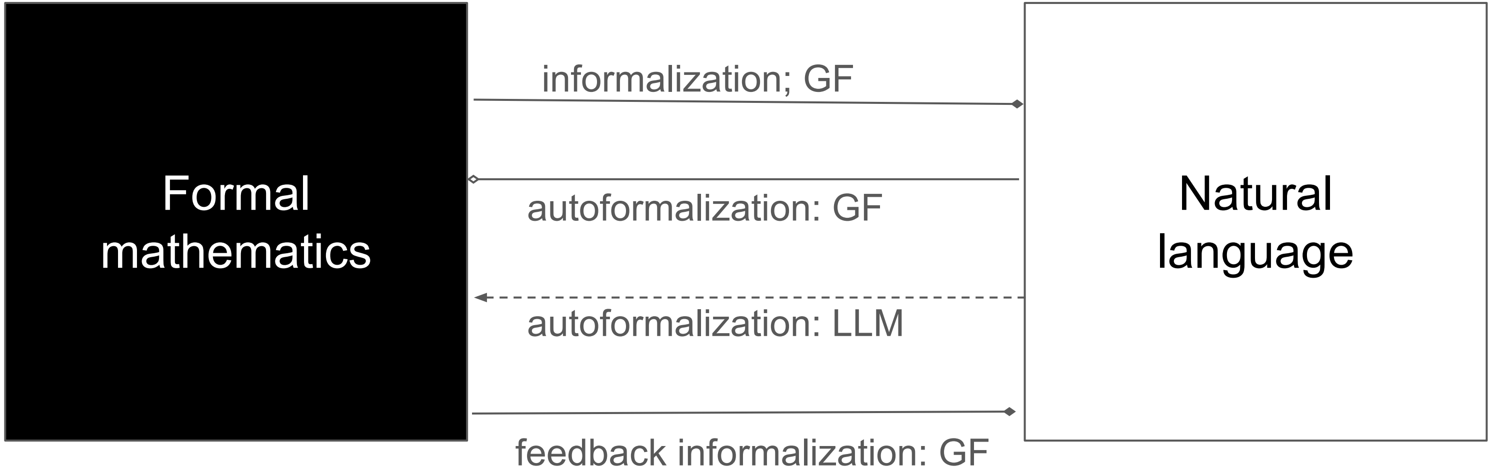

Agda
Rocq
Lean
formal
Dedukti
informalization
formalization
informal
Informath.gf
English
French
Swedish
to one
to many
total
partial


even 4
not (even 5)
informalization; GF
4 is even
5 is not even
autoformalization: GF


even 4
not (even 5)
NO PARSE
informalization; GF
autoformalization: GF
4 is even
5 is not even
5 can't be even


even 4
not (even 5)
NO PARSE
even 5
informalization; GF
autoformalization: GF
autoformalization: LLM
4 is even
5 is not even
5 can't be even


even 4
not (even 5)
NO PARSE
even 5
informalization; GF
autoformalization: GF
autoformalization: LLM
feedback informalization: GF
4 is even
5 is not even
5 can't be even
5 is even



Vision:

- the formal system is a black box that performs verification
- humans communicate with it in natural language

Mapping between Dedukti and GF

```
-- Dedukti.bnf

MJmts. Module ::= [Jmt] ;

terminator Jmt "" ;

comment "(;" ";)" ;
comment "#" ; ----

JStatic.  Jmt ::= QIdent ":" Exp "." ;
JDef.     Jmt ::= "def" QIdent MTyp MExp "." ;
JInj.     Jmt ::= "inj" QIdent MTyp MExp "." ;
JThm.     Jmt ::= "thm" QIdent MTyp MExp "." ;
JRules.   Jmt ::= [Rule] "." ;

RRule.  Rule ::= "[" [Pattbind] "]" Patt "-->" Exp ;
separator nonempty Rule "" ;

separator Pattbind "," ;

MTNone. MTyp ::= ;
MTExp.  MTyp ::= ":" Exp ;

MENone. MExp ::= ;
MEExp.  MExp ::= ":=" Exp ;

EIdent.  Exp9 ::= QIdent ;
EApp.    Exp5 ::= Exp5 Exp6 ;
EAbs.    Exp2 ::= Bind "=>" Exp2 ;
EFun.    Exp1 ::= Hypo "->" Exp1 ;

coercions Exp 9 ;

-- plus some rules for Hypo and Bind

token QIdent (letter | digit | '_' | '!' | '?' | '\'')+
('.' (letter | digit | '_' | '!' | '?' | '\'')+)? ;
```

```
-- Dedukti.bnf

MJmts. Module ::= [Jmt] ;

terminator Jmt "" ;

comment "(;" ";)" ;
comment "#" ; ----

JStatic.  Jmt ::= QIdent ":" Exp "." ;
JDef.     Jmt ::= "def" QIdent MTyp MExp "." ;
JInj.     Jmt ::= "inj" QIdent MTyp MExp "." ;
JThm.     Jmt ::= "thm" QIdent MTyp MExp "." ;
JRules.   Jmt ::= [Rule] "." ;

RRule.  Rule ::= "[" [Pattbind] "]" Patt "-->" Exp ;
separator nonempty Rule "" ;

separator Pattbind "," ;

MTNone. MTyp ::= ;
MTExp.  MTyp ::= ":" Exp ;

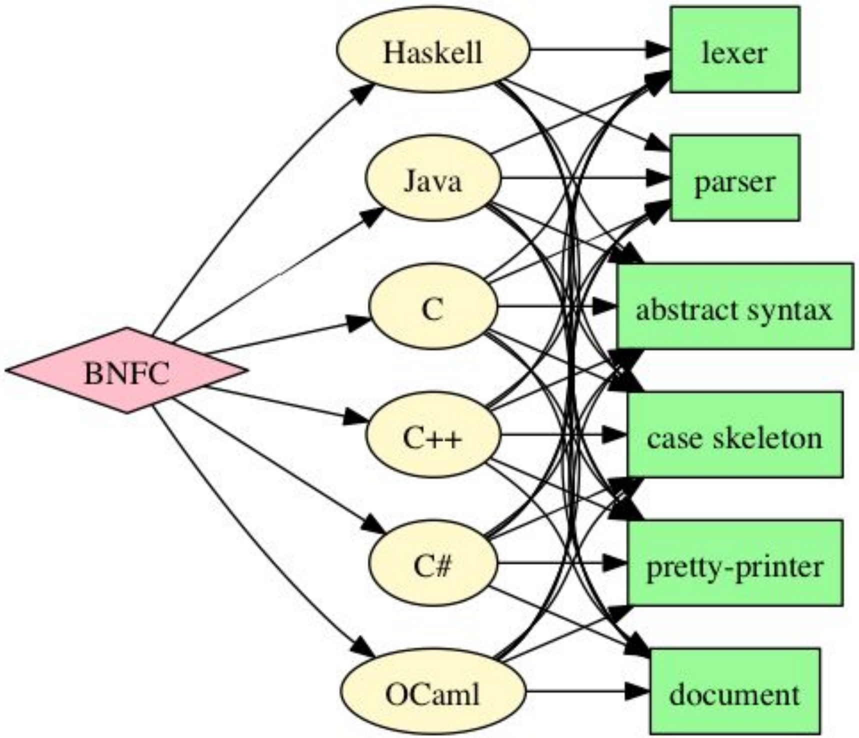
MENone. MExp ::= ;
MEExp.  MExp ::= ":=" Exp ;

EIdent.  Exp9 ::= QIdent ;
EApp.    Exp5 ::= Exp5 Exp6 ;
EAbs.    Exp2 ::= Bind "=>" Exp2 ;
EFun.    Exp1 ::= Hypo "->" Exp1 ;

coercions Exp 9 ;

-- plus some rules for Hypo and Bind

token QIdent (letter | digit | '_' | '!' | '?' | '\'')+
('.' (letter | digit | '_' | '!' | '?' | '\'')+)? ;
```



https://bnfc.digitalgrammars.com/

```
-- Dedukti.bnf

MJmts. Module ::= [Jmt] ;

terminator Jmt "" ;

comment "(;" ";)" ;
comment "#" ; ----

JStatic.  Jmt ::= QIdent ":" Exp "." ;
JDef.     Jmt ::= "def" QIdent MTyp MExp "." ;
JInj.     Jmt ::= "inj" QIdent MTyp MExp "." ;
JThm.     Jmt ::= "thm" QIdent MTyp MExp "." ;
JRules.   Jmt ::= [Rule] "." ;

RRule.  Rule ::= "[" [Pattbind] "]" Patt "-->" Exp ;
separator nonempty Rule "" ;

separator Pattbind "," ;

MTNone. MTyp ::= ;
MTExp.  MTyp ::= ":" Exp ;

MENone. MExp ::= ;
MEExp.  MExp ::= ":=" Exp ;

EIdent.  Exp9 ::= QIdent ;
EApp.    Exp5 ::= Exp5 Exp6 ;
EAbs.    Exp2 ::= Bind "=>" Exp2 ;
EFun.    Exp1 ::= Hypo "->" Exp1 ;

coercions Exp 9 ;

-- plus some rules for Hypo and Bind

token QIdent (letter | digit | '_' | '!' | '?' | '\'')+
('.' (letter | digit | '_' | '!' | '?' | '\'')+)? ;
```

```
-- MathCore.gf

abstract MathCore =
  Terms, UserConstants
  ** {
cat
  Jmt ;
  Exp ;
  Exps ;
  Prop ;
  Kind ;
  Hypo ;
  [Hypo] ;
  Proof ;
  Label ;
  -- plus more categories
fun
  ThmJmt : Label -> [Hypo] -> Prop -> Proof -> Jmt ;
  AxiomJmt : Label -> [Hypo] -> Prop -> Jmt ;
  DefPropJmt : Label -> [Hypo] -> Prop -> Prop -> Jmt ;
  DefKindJmt : Label -> [Hypo] -> Kind -> Kind -> Jmt ;
  DefExpJmt  : Label -> [Hypo] -> Exp -> Kind -> Exp -> Jmt ;
  AxiomPropJmt : Label -> [Hypo] -> Prop -> Jmt ;
  AxiomKindJmt : Label -> [Hypo] -> Kind -> Jmt ;
  AxiomExpJmt  : Label -> [Hypo] -> Exp -> Kind -> Jmt ;

  AppExp : Exp -> Exps -> Exp ;
  AbsExp : [Ident] -> Exp -> Exp ;
  TermExp : Term -> Exp ;
  KindExp : Kind -> Exp ;
  TypedExp : Exp -> Kind -> Exp ;

  AndProp : [Prop] -> Prop ;
  OrProp : [Prop] -> Prop ;
  IfProp : Prop -> Prop -> Prop ;
  IffProp : Prop -> Prop -> Prop ;
  NotProp : Prop -> Prop ;
  -- plus many more functions
```

```
-- Dedukti.bnf

MJmts. Module ::= [Jmt] ;

terminator

comment "(
comment "#

JStatic.
JDef.
JInj.
JThm.
JRules.

RRule.   Ru
separator

separator

MTNone.  MT
MTExp.   MT

MENone.  ME
MEExp.   ME

EIdent.   E
EApp.     E
EAbs.     E
EFun.     E

coercions

-- plus so

token QIde
('.' (letter | digit | '_' | '!' | '?' | '\'')+)? ;
```

```
module AbsDedukti where

data Tree (a :: Tag) where
    MJmts :: [Jmt] -> Tree 'Module_
    JStatic :: QIdent -> Exp -> Tree 'Jmt_
    JDef :: QIdent -> MTyp -> MExp -> Tree 'Jmt_
    JInj :: QIdent -> MTyp -> MExp -> Tree 'Jmt_
    JThm :: QIdent -> MTyp -> MExp -> Tree 'Jmt_
    JRules :: [Rule] -> Tree 'Jmt_
    RRule :: [Pattbind] -> Patt -> Exp -> Tree 'Rule_
    MTNone :: Tree 'MTyp_
    MTExp :: Exp -> Tree 'MTyp_
    MENone :: Tree 'MExp_
    MEExp :: Exp -> Tree 'MExp_
    EIdent :: QIdent -> Tree 'Exp_
    EApp :: Exp -> Exp -> Tree 'Exp_
    EAbs :: Bind -> Exp -> Tree 'Exp_
    EFun :: Hypo -> Exp -> Tree 'Exp_
    BVar :: QIdent -> Tree 'Bind_
    BTyped :: QIdent -> Exp -> Tree 'Bind_
    PBVar :: QIdent -> Tree 'Pattbind_
    PBTyped :: QIdent -> Exp -> Tree 'Pattbind_
    HExp :: Exp -> Tree 'Hypo_
    HVarExp :: QIdent -> Exp -> Tree 'Hypo_
    HParVarExp :: QIdent -> Exp -> Tree 'Hypo_
    PVar :: QIdent -> Tree 'Patt_
    PBracket :: Patt -> Tree 'Patt_
    PApp :: Patt -> Patt -> Tree 'Patt_
    PBind :: Bind -> Patt -> Tree 'Patt_
    QIdent :: String -> Tree 'QIdent_
```

```
-- MathCore.gf

abstract MathCore =
  Terms, UserConstants
  ** {
cat
  Jmt ;
  Exp ;
  Exps ;
  Prop ;
  Kind ;
  Hypo ;
  [Hypo] ;
  Proof ;
  Label ;
  -- plus more categories
fun
  ThmJmt : Label -> [Hypo] -> Prop -> Proof -> Jmt ;
  AxiomJmt : Label -> [Hypo] -> Prop -> Jmt ;
  DefPropJmt : Label -> [Hypo] -> Prop -> Prop -> Jmt ;
  DefKindJmt : Label -> [Hypo] -> Kind -> Kind -> Jmt ;
  DefExpJmt  : Label -> [Hypo] -> Exp -> Kind -> Exp -> Jmt ;
  AxiomPropJmt : Label -> [Hypo] -> Prop -> Jmt ;
  AxiomKindJmt : Label -> [Hypo] -> Kind -> Jmt ;
  AxiomExpJmt  : Label -> [Hypo] -> Exp -> Kind -> Jmt ;

  AppExp : Exp -> Exps -> Exp ;
  AbsExp : [Ident] -> Exp -> Exp ;
  TermExp : Term -> Exp ;
  KindExp : Kind -> Exp ;
  TypedExp : Exp -> Kind -> Exp ;

  AndProp : [Prop] -> Prop ;
  OrProp : [Prop] -> Prop ;
  IfProp : Prop -> Prop -> Prop ;
  IffProp : Prop -> Prop -> Prop ;
  NotProp : Prop -> Prop ;
  -- plus quite a few more functions
```

```
-- Dedukti.bnf

MJmts. Module ::= [Jmt] ;

terminator

comment "(
comment "#

JStatic.
JDef.
JInj.
JThm.
JRules.

RRule.  Ru
separator

separator

MTNone. MT
MTExp.  MT

MENone. ME
MEExp.  ME

EIdent.  E
EApp.    E
EAbs.    E
EFun.    E

coercions

-- plus so

token QIde
('.' (letter | digit | '_' | '!' | '?' | '\'')+)? ;
```

```
module AbsDedukti where

data Tree (a :: Tag) where
    MJmts :: [Jmt] -> Tree 'Module_
    JStatic :: QIdent -> Exp -> Tree 'Jmt_
    JDef :: QIdent -> MTyp -> MExp -> Tree 'Jmt_
    JInj :: QIdent -> MTyp -> MExp -> Tree 'Jmt_
    JThm :: QIdent -> MTyp -> MExp -> Tree 'Jmt_
    JRules :: [Rule] -> Tree 'Jmt_
    RRule :: [Pattbind] -> Patt -> Exp -> Tree 'Rule_
    MTNone :: Tree 'MTyp_
    MTExp :: Exp -> Tree 'MTyp_
    MENone :: Tree 'MExp_
    MEExp :: Exp -> Tree 'MExp_
    EIdent :: QIdent -> Tree 'Exp_
    EApp :: Exp -> Exp -> Tree 'Exp_
    EAbs :: Bind -> Exp -> Tree 'Exp_
    EFun :: Hypo -> Exp -> Tree 'Exp_
    BVar :: QIdent -> Tree 'Bind_
    BTyped :: QIdent -> Exp -> Tree 'Bind_
    PBVar :: QIdent -> Tree 'Pattbind_
    PBTyped :: QIdent -> Exp -> Tree 'Pattbind_
    HExp :: Exp -> Tree 'Hypo_
    HVarExp :: QIdent -> Exp -> Tree 'Hypo_
    HParVarExp :: QIdent -> Exp -> Tree 'Hypo_
    PVar :: QIdent -> Tree 'Patt_
    PBracket :: Patt -> Tree 'Patt_
    PApp :: Patt -> Patt -> Tree 'Patt_
    PBind :: Bind -> Patt -> Tree 'Patt_
    QIdent :: String -> Tree 'QIdent_
-- this is all
```

```
-- MathCore.gf

abstract MathCore =
  Terms, UserConstants

t ;
t ;
-> Jmt ;

mt ;

  NotProp : Prop -> Prop ;
  -- plus more functions
```

```
module Informath where

data Tree :: * -> * where
  GAndAdj :: GListAdj -> Tree GAdj_
  GComparAdj :: GCompar -> GExp -> Tree GAdj_
  GOrAdj :: GListAdj -> Tree GAdj_
  GReladjAdj :: GReladj -> GExp -> Tree GAdj_
  LexAdj :: String -> Tree GAdj_
  GIdentsArgKind :: GKind -> GListIdent -> Tree GArgKind_
  GKindArgKind :: GKind -> Tree GArgKind_
  LexCompar :: String -> Tree GCompar_
  LexComparnoun :: String -> Tree GComparnoun_
  LexConst :: String -> Tree GConst_
  GComparEqsign :: GCompar -> Tree GEqsign_
  GComparnounEqsign :: GComparnoun -> Tree GEqsign_
  GEBinary :: GEqsign -> GTerm -> GTerm -> Tree GEquation_
  GAbsExp :: GListIdent -> GExp -> Tree GExp_
  GAllIdentsKindExp :: GListIdent -> GKind -> Tree GExp_
  GAllKindExp :: GKind -> Tree GExp_
  GAndExp :: GListExp -> Tree GExp_
  GAppExp :: GExp -> GExps -> Tree GExp_
  GCoercionExp :: GCoercion -> GExp -> Tree GExp_
  GConstExp :: GConst -> Tree GExp_
  GEveryIdentKindExp :: GIdent -> GKind -> Tree GExp_
  GEveryKindExp :: GKind -> Tree GExp_
  GFunListExp :: GFun -> GExps -> Tree GExp_
  GIndefIdentKindExp :: GIdent -> GKind -> Tree GExp_
  GIndefKindExp :: GKind -> Tree GExp_
  GIndexedTermExp :: GInt -> Tree GExp_

-- plus quite a few more
```

```
module Dedukti2Core where

import Dedukti.AbsDedukti
import Informath
import DeduktiOperations

jmt2jmt :: Jmt -> GJmt
jmt2jmt jmt = case jmt of
  JDef ident (MTExp typ) meexp ->
    let mexp = case meexp of
          MEExp exp -> Just exp
          _ -> Nothing
    in case (splitType typ, guessCat ident typ) of
      ((hypos, kind), c) | elem c ["Noun", "Set"] ->
          (maybe (GAxiomKindJmt axiomLabel)
               (\exp x y -> GDefKindJmt definitionLabel x y (exp2kind exp)) mexp)
            (GListHypo (hypos2hypos hypos))
            (ident2kind ident)
      ((hypos, kind), c) | elem c ["Name", "Const", "Unknown"] ->
          (maybe (GAxiomExpJmt axiomLabel)
                  (\exp x y z -> GDefExpJmt definitionLabel x y z (exp2exp exp)) mexp)
            (GListHypo (hypos2hypos hypos))
            (ident2exp ident)
            (exp2kind kind)

…

exp2kind :: Exp -> GKind

exp2prop :: Exp -> GProp

exp2exp :: Exp -> GExp

exp2proof :: Exp -> GProof
```

```
-- Dedukti.bnf

MJmts. Module ::= [Jmt]

terminator

comment "(
comment "#

JStatic.
JDef.
JInj.
JThm.
JRules.

RRule.   Ru
separator

separator

MTNone. MT
MTExp.  MT

MENone. ME
MEExp.  ME

EIdent.  E
EApp.    E
EAbs.    E
EFun.    E

coercions

-- plus so

token QIde
('.' (letter | digit |
```

```
module AbsDe

data Tree (a
    MJmts ::
    JStatic
    JDef ::
    JInj ::
    JThm ::
    JRules :
    RRule ::
    MTNone :
    MTExp ::
    MENone :
    MEExp ::
    EIdent :
    EApp ::
    EAbs ::
    EFun ::
    BVar ::
    BTyped :
    PBVar ::
    PBTyped
    HExp ::
    HVarExp
    HParVarE
    PVar ::
    PBracket
    PApp ::
    PBind ::
    QIdent :
```

```
GAdj_

GAdj_

-> Tree GArgKind_
_

rnoun_

gn_
ee GEqsign_
-> Tree GEquation_
GExp_
nd -> Tree GExp_

ree GExp_

-> Tree GExp_

Exp_
-> Tree GExp_
```

```
t ;
t ;
 -> Jmt ;

mt ;
```

Dedukti Exp	GF category	linearization	linguistic category
`union A B`	`Exp`	*the union of A and B*	noun phrase
`Nat`	`Kind`	*natural number*	common noun
`divisible 9 3`	`Prop`	*9 is divisible by 3*	sentence
`oddS 0 evenZ`	`Proof`	*0 is even. Therefore 1 is odd.*	text

```
-- Dedukti.bnf

MJmts. Module ::= [Jmt]

terminator

comment "(
comment "#

JStatic.
JDef.
JInj.
JThm.
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EIdent.  E
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coercions

-- plus so

token QIde
('.' (letter | digit |
```

```
module AbsDe

data Tree (a
     MJmts ::
     JStatic
     JDef ::
     JInj ::
     JThm ::
     JRules :
     RRule ::
     MTNone :
     MTExp ::
     MENone :
     MEExp ::
     EIdent :
     EApp ::
     EAbs ::
     EFun ::
     BVar ::
     BTyped :
     PBVar ::
     PBTyped
     HExp ::
     HVarExp
     HParVarE
     PVar ::
     PBracket
     PApp ::
     PBind ::
     QIdent :
```

```
module Dedukti2Core where

impo
impo
impo

jmt2
jmt2
  JD

  JD

…

exp2

exp2

exp2

exp2

iden
iden
  QI
```

```
module Core2Dedukti where

import Dedukti.AbsDedukti
import Informath
import DeduktiOperations

prop2dedukti :: GProp -> Exp
prop2dedukti prop = case prop of
  GProofProp p -> EApp (EIdent (QIdent "Proof")) (prop2dedukti p)
  GFalseProp -> propFalse
  GIdentProp ident -> EIdent (ident2ident ident)
  GAndProp (GListProp props) -> foldl1 propAnd (map prop2dedukti props)

kind2dedukti :: GKind -> Exp
kind2dedukti kind = case kind of
  GElemKind k -> EApp (EIdent (QIdent "Elem")) (kind2dedukti k)
  GTermKind (GTIdent ident) -> EIdent (ident2ident ident)
  GSetKind (LexSet s) -> EIdent (QIdent (s))

exp2dedukti :: GExp -> Exp
exp2dedukti exp = case exp of
  GTermExp (GTNumber (GInt n)) -> int2exp n
  GTermExp (GTIdent ident) -> EIdent (ident2ident ident)
  GAppExp exp exps ->
    foldl1 EApp (map exp2dedukti (exp : (exps2list exps)))
  GAbsExp (GListIdent idents) exp ->
    foldr
      (\x y -> EAbs (BVar (ident2ident x)) y)
      (exp2dedukti exp)
      idents
```

```
e GArgKind_

sign_             t ;
e GEquation_      t ;
                   -> Jmt ;
Tree GExp_

                  mt ;

xp_

e GExp_

e GExp_
```

Dealing with identifiers

```
-- BaseConstants.dk

Set : Type.
Prop : Type.

(; ignored in Dedukti2Core ;)
Elem : Set -> Type.
Proof : Prop -> Type.

(; logical operators, hard-coded in MathCore ;)
false : Prop.
and : (A : Prop) -> (B : Prop) -> Prop.
or : (A : Prop) -> (B : Prop) -> Prop.
if : Prop -> Prop -> Prop.
forall : (A : Set) -> (Elem A -> Prop) -> Prop.
exists : (A : Set) -> (Elem A -> Prop) -> Prop.

def not : Prop -> Prop := A => if A false.
def iff : Prop -> Prop -> Prop :=
  A => B => and (if A B) (if B A).

(; constants defined in a lexicon ;)

def Nat : Set := Num.
def Int : Set := Num.
def Rat : Set := Num.
def Real : Set := Num.

Eq : Elem Real -> Elem Real -> Prop.
Lt : Elem Real -> Elem Real -> Prop.
Gt : Elem Real -> Elem Real -> Prop.
Neq : Elem Real -> Elem Real -> Prop.
Leq : Elem Real -> Elem Real -> Prop.
Geq : Elem Real -> Elem Real -> Prop.

plus : (x : Elem Real) -> (y : Elem Real) -> Elem Real.
minus : Elem Real -> Elem Real -> Elem Real.
times : Elem Real -> Elem Real -> Elem Real.
```

```
(; BaseConstants.dk ;)

(; constants defined in a lexicon ;)

Nat : Set.
Int : Set.
Rat : Set.
Real : Set.

Eq : Elem Real -> Elem Real -> Prop.
Lt : Elem Real -> Elem Real -> Prop.
Gt : Elem Real -> Elem Real -> Prop.

plus : (x : Elem Real) -> (y : Elem Real) -> Elem Real.
minus : Elem Real -> Elem Real -> Elem Real.
times : Elem Real -> Elem Real -> Elem Real.

even : Elem Int -> Prop.
def odd : Elem Int -> Prop := n => not (even n).
```

```
# base_constant_data.dkgf

# for translating between Dedukti and GF abstract syntax

Nat BASE Set natural_Set
Int BASE Set integer_Set
Rat BASE Set rational_Set
Real BASE Set real_Set

Eq BASE Compar Eq_Compar
Lt BASE Compar Lt_Compar
Gt BASE Compar Gt_Compar

plus BASE Oper plus_Oper
minus BASE Oper minus_Oper
times BASE Oper times_Oper

even BASE Adj even_Adj
odd BASE Adj odd_Adj

# for generating GF linearization rules

#LIN Eng natural_Set = mkSet "N" "natural" number_N
#LIN Fre natural_Set = mkSet L.natural_Set "naturel" nombre_N
#LIN Swe natural_Set = mkSet L.natural_Set "naturlig" tal_N

#LIN Eng Lt_Compar = mkCompar "<" "less" "than"
#LIN Fre Lt_Compar = mkCompar "<" (mkAP (mkA "inférieur")) dative
#LIN Swe Lt_Compar = mkCompar "<" "mindre" "än"

#LIN Eng even_Adj = mkAdj "even"
#LIN Fre even_Adj = mkAdj "pair"
#LIN Swe even_Adj = mkAdj "jämn"

# for converting identifiers from third-party projects

le ALIAS matita Leq
```

```
abstract BaseConstants = {

-- GF cat           usage                                example
-----------------------------------------------------------------------
  Noun ;          -- Kind                             -- set
  Fam ;           -- Kind -> Kind                     -- list of integers
  Adj ;           -- Exp -> Prop                      -- even
  Verb ;          -- Exp -> Exp                       -- converge
  Reladj ;        -- Exp -> Exp -> Prop               -- divisible by
  Relverb ;       -- Exp -> Exp -> Prop               -- divide
  Relnoun ;       -- Exp -> Exp -> Prop               -- root of
  Name ;          -- Exp                              -- contradiction
  Fun ;           -- [Exp] -> Exp                     -- radius of
  Label ;         -- Exp                              -- theorem 1

  Set ;           -- Kind | Term                      -- integer, Z
  Const ;         -- Exp | Term                       -- the empty set, Ø
  Oper ;          -- Exp -> Exp -> Exp  | Term        -- the sum of, +
  Compar ;        -- Exp -> Exp -> Prop | Formula     -- greater than, >
  Comparnoun ;    -- Exp -> Exp -> Prop | Formula     -- a subset of, \sub
```

```
def sphenic : Nat -> Prop
  := ...
(; GF: sphenic number ;)
```

lexical rule extraction

```
# from Wikidata

{"Q638185": {
  "pl": "Liczby sfeniczne",
  "de": "sphenische Zahl",
  "en": "sphenic number",
  "es": "número esfénico",
  "fr": "nombre sphénique",
  "zh": "楔形数",
  "sv": "sfeniskt tal",
  "ta": "ஸ்ஃபீனிக் எண்",
  }
}
```

```
sphenic NEW number_theory Adj spenic_Adj

#LIN Eng sphenic_Adj = mkAdj "sphenic"
#LIN Fre sphenic_Adj = mkAdj "sphénique"
#LIN Swe sphenic_Adj = mkAdj "sfenisk"
```

AR, Building Grammar Libraries for Mathematics and Avoiding Manual Work.. Presentation at Hausdorff Center for Mathematics, 2024, https://www.youtube.com/watch?v=EQ-k_JQ7fDM&t=5s

Lexicon Extraction from Wikidata

Ingredients

Wikidata: https://www.wikidata.org

- a list of math terms provided by Frederik Schaeffer
- MathGloss of Lucy Horowitz and Valeria de Paiva
- in total, 5381 terms

GF RGL

- smart paradigms for inflection
- syntactic combination rules
- morphological dictionaries

UD parsing

- extract parts of speech, lemmas, and some inflection for unknown words

Extraction functions for syntax (using the RGL)

```
AdjCN : AP -> CN -> CN ;               -- continuous function
CompoundN : N -> N -> N ;              -- function space
IntCompoundCN : Int -> CN -> CN ;      -- 13-cube
NameCompoundCN : PN -> CN -> CN ;      -- Lie group
NounIntCN : CN -> Int -> CN ;          -- Grinberg graph 42
NounPrepCN : CN -> Adv -> CN ;         -- ring of sets
NounGenCN : CN -> NP -> CN ;           -- bishop's graph

PositA : A -> AP ;                     -- uniform
AdAP : AdA -> AP -> AP ;               -- almost uniform
AAdAP : A -> AP -> AP ;                -- algebraically closed
PastPartAP : V -> AP ;                 -- connected

PrepNP : Prep -> NP -> Adv ;           -- (integration) by parts

-- plus some more functions, 21 functions in total
```

RGL morphological dictionaries

```
lin isotropy_N = mkN "isotropy" "isotropies" ;
lin israeli_A = mkAMost "israeli" "israelily" ;
lin israeli_N = mkN "israeli" "israelis" ;
lin issue_N = mkN "issue" "issues" ;
lin issue_V = mkV "issue" "issued" "issued" ;
lin issuer_N = mkN "issuer" "issuers" ;
lin isthmian_A = mkAMost "isthmian" "isthmianly" ;
lin isthmus_N = mkN "isthmus" "isthmuses" ;
lin italic_A = mkAMost "italic" "italicly" ;
lin italic_N = mkN "italic" "italics" ;
lin italicize_V = mkV "italicize" "italicized" "italicized" ;
lin itch_N = mkN "itch" "itches" ;
lin itch_V = mkV "itch" "itched" "itched" ;
lin itchy_A = mkA "itchy" "itchier" "itchiest" "itchily" ;
lin item_Adv = mkAdv "item" ;
lin item_N = mkN "item" "items" ;
lin itemize_V = mkV "itemize" "itemized" "itemized" ;
lin iterate_V = mkV "iterate" "iterated" "iterated" ;
lin iteration_N = mkN "iteration" "iterations" ;
lin iterative_A = mkAMost "iterative" "iteratively" ;

-- English: 56,598 lemmas in total
```

```
lin abfieseln_V = prefixV "ab" (regV "fieseln") ;
lin abfinden_V = prefixV "ab" (irregV "finden" "findet" "fand"
"fände" "gefunden") ;
lin abfindung_N = mkN "Abfindung" ;
lin abflachen_V = prefixV "ab" (regV "flachen") ;
lin abflauen_V = prefixV "ab" (regV "flauen") ;
lin abfliegen_V = prefixV "ab" (irregV "fliegen" "fliegt" "flog"
"flöge" "geflogen") ;
lin abfliessen_V = prefixV "ab" (irregV "fließen" "fließt" "floss"
"floss" "geflossen") ;
lin abflug_N = mkN "Abflug" "Abflüge" masculine ;
lin abfluss_N = mkN "Abfluss" "Abflüsse" masculine ;
lin abflusslos_A = regA "abflusslos" ;
lin abflussrohr_N = mkN "Abflussrohr" "Abflussrohre" neuter ;
lin abfolge_N = mkN "Abfolge" "Abfolgen" feminine ;
lin abformen_V = prefixV "ab" (regV "formen") ;
lin abformmasse_N = mkN "Abformmasse" "Abformmassen" feminine ;
lin abfotografieren_V = prefixV "ab" (regV "fotografieren") ;
lin abfrage_N = mkN "Abfrage" "Abfragen" feminine ;
lin abfragen_V = prefixV "ab" (regV "fragen") ;

-- German: 44,229 lemmas in total
```

Demo: building a lexicon for French

```
./build_lexicon.py (-first|-added) <fr> <Fre> -from=<Eng>? <STEPNUM>+
```

- Step 0: preparations
- Step 1: extract wikidata for that language into qlist
- Step 2: parse with UDPipe
- Step 3: use the UDPipe parse to clean up corpus and add to lexicon
- Step 4: build a lexicon extension
- Step 5: parse the terms with the extended lexicon
- Step 6: (if -first) generate GF modules for abstract and the first concrete
- Step 7: (if -add) add a new concrete syntax
- Step 8: test your grammar in GF

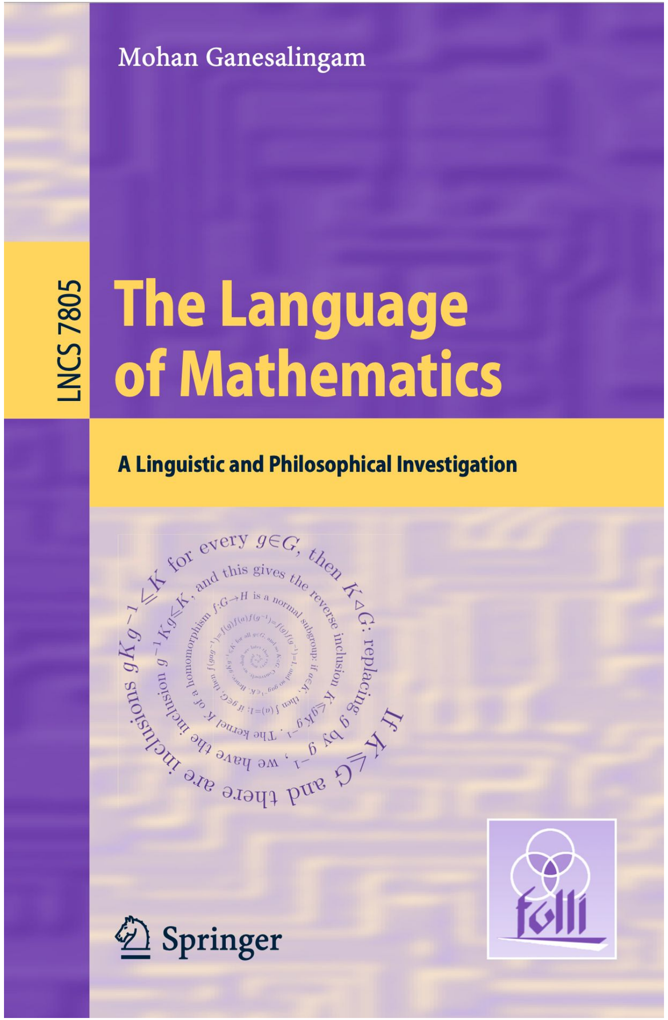
https://github.com/aarneranta/informath/tree/main/old/v2/extraction

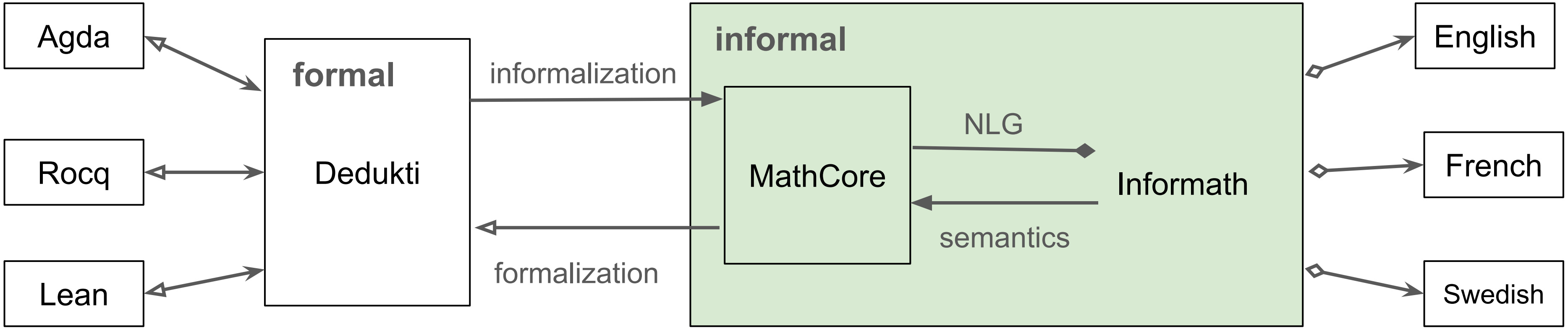
From MathCore to full Informath

has

natural language content that ~~lacks~~ the inherent diversity and flexibility in expression: they are ~~rigid and not~~ natural-language-like.

natural language content that ~~lacks~~ *has* the inherent diversity and flexibility in expression: they are ~~rigid and not~~ natural-language-like.





	to one	to many
total		
partial		

```
prop110 : (a : Elem Int) -> (c : Elem Int) ->
 Proof (and (odd a) (odd c)) -> Proof (forall
 Int (b => even (plus (times a b) (times b c)))).
```

Prop110. For all instances a and c of integers, if we can prove that a is odd and c is odd, then we can prove that for all integers b, the sum of the product of a and b and the product of b and c is even.

```
abstract Informath = MathCore ** {


fun
-- use symbolic expressions whenever possible
  FormulaProp : Formula -> Prop ;
  SetTerm : Set -> Term ;
  ConstTerm : Const -> Term ;
  ComparEqsign : Compar -> Eqsign ;


-- aggregation

  AndAdj : [Adj] -> Adj ;
  OrAdj : [Adj] -> Adj ;

  AndExp : [Exp] -> Exp ;
  OrExp : [Exp] -> Exp ;


-- post-quantification

  PostQuantProp : Prop -> Exp -> Prop ;


}
```

```
prop110 : (a : Elem Int) -> (c : Elem Int) ->
 Proof (and (odd a) (odd c)) -> Proof (forall
 Int (b => even (plus (times a b) (times b c)))).
```

Prop110. For all instances a and c of integers, if we can prove that a is odd and c is odd, then we can prove that for all integers b, the sum of the product of a and b and the product of b and c is even.

Prop110. Let $a, c \in Z$. Assume that both a and c are odd. Then for all integers b, $ab + bc$ is even.

Prop110. Let $a, c \in Z$. Assume that both a and c are odd. Then $ab + bc$ is even for all integers b.

```haskell
module Core2Informath where

import Informath

nlg :: Opts -> Tree a -> [Tree a]
nlg opts tree = concatMap variations [t, ut, ft, aft, iaft, viaft]
 where
   t = unparenth tree
   ut = uncoerce t
   ft = formalize ut
   aft = aggregate (flatten ft)
   iaft = insitu aft
   viaft = varless iaft


insitu :: Tree a -> Tree a
insitu t = case t of
  GAllProp (GListArgKind [argkind]) (GAdjProp adj exp) -> case subst argkind exp of
    Just (x, kind) -> GAdjProp adj (GAllIdentsKindExp (GListIdent [x]) kind)
    _ -> t
  GAllProp (GListArgKind [argkind]) (GNotAdjProp adj exp) -> case subst argkind exp of
    Just (x, kind) -> GAdjProp adj (GNoIdentsKindExp (GListIdent [x]) kind)
    _ -> t
  GExistProp (GListArgKind [argkind]) (GAdjProp adj exp) -> case subst argkind exp of
    Just (x, kind) -> GAdjProp adj (GSomeIdentsKindExp (GListIdent [x]) kind)
    _ -> t
  _ -> composOp insitu t


varless :: Tree a -> Tree a
varless t = case t of
  GAllIdentsKindExp (GListIdent [_]) kind -> GAllKindExp kind
  GNoIdentsKindExp (GListIdent [_]) kind -> GNoKindExp kind
  GSomeIdentsKindExp (GListIdent [_]) kind -> GSomeKindExp kind
  _ -> composOp varless t
```

NLG (Natural Language Generation) is a combination of selected **almost compositional operations.**

Another example: **in situ quantification**

$$(Q\ x : A)B(x) \ \Rightarrow B(Q\ A)$$

if x occurs exactly once in B:

The variable can optionally be omitted.

B Bringert and A. Ranta, A pattern for almost compositional functions. Journal of Functional Programming 18 (5-6), 567-598, 2008.

```
abstract Informath = MathCore ** {

cat
  [Adj] {2} ;
  [Exp] {2} ;

fun
-- to use symbolic expressions whenever possible
  FormulaProp : Formula -> Prop ;
  SetTerm : Set -> Term ;
  ConstTerm : Const -> Term ;
  ComparEqsign : Compar -> Eqsign ;

-- to remove parentheses around complex propositions
  SimpleAndProp : [Prop] -> Prop ;

-- to aggregate adjectives and expressions
  AndAdj : [Adj] -> Adj ;
  OrAdj : [Adj] -> Adj ;

  AndExp : [Exp] -> Exp ;
  OrExp : [Exp] -> Exp ;

-- in situ quantifiers

  AllKindExp : Kind -> Exp ;
  AllIdentsKindExp : [Ident] -> Kind -> Exp ;

  SomeKindExp : Kind -> Exp ;
  SomeIdentsKindExp : [Ident] -> Kind -> Exp ;

  NoIdentsKindExp : [Ident] -> Kind -> Exp ;
  NoKindExp : Kind -> Exp ;

-- miscellaneous alternative expressions

  PostQuantProp : Prop -> Exp -> Prop ;
}
```

```
prop50 : Proof (forall Nat
  (n => not (and (even n) (odd n)))).
```

Prop50. We can prove that for all natural numbers n, it is not the case that n is even and n is odd.

Prop50. For all natural numbers n, n is not both even and odd.

Prop50. No natural number n is both even and odd.

Prop50. No natural number is both even and odd.

Scoring and ranking alternative phrases

```
data Scores = Scores {
  tree_length :: Int,
  tree_depth :: Int,
  characters :: Int,
  tokens :: Int,
  subsequent_dollars :: Int,
  initial_dollars :: Int,
  parses :: Int
  }
```

```
$ ./RunInformath -ranking -variations -test-ambiguity test/prop110.dk
## showing a sample from 87 results, first and last included

Prop110. Let $a , c \in Z$. Then if $a$ and $c$ are odd, then $a b + b c$ is even for every integer $b$.
%% (Scores {tree_length = 55, tree_depth = 10, characters = 104, tokens = 40, subsequent_dollars = 0, initial_dollars =
0, parses = 2},211)

Prop110. Let $a , c \in Z$. Then $a$ and $c$ are odd, only if $a b + b c$ is even for every integer $b$.
%% (Scores {tree_length = 59, tree_depth = 10, characters = 110, tokens = 43, subsequent_dollars = 1, initial_dollars =
0, parses = 2},225)

Prop110. Let $a$ and $c$ be integers. Assume that $a$ and $c$ are odd. Then $a b + b c$ is even for every integer $b$.
%% (Scores {tree_length = 53, tree_depth = 11, characters = 118, tokens = 42, subsequent_dollars = 0, initial_dollars =
0, parses = 1},225)

Prop110. Let $a$ and $c$ be integers. Assume that $a$ and $c$ are odd. Then for all integers $b$, $a b + b c$ is even.
%% (Scores {tree_length = 55, tree_depth = 11, characters = 118, tokens = 43, subsequent_dollars = 1, initial_dollars =
0, parses = 1},229)

Prop110. For all integers $a$ and $c$, if $a$ is odd and $c$ is odd, then for all integers $b$, $a b + b c$ is even.
%% (Scores {tree_length = 57, tree_depth = 11, characters = 116, tokens = 44, subsequent_dollars = 1, initial_dollars =
0, parses = 1},230)

Prop110. Let $a$ and $c$ be instances of integers. Then we can prove that $a$ is odd and $c$ is odd, only if we can
prove that for all integers $b$, the sum of the product of $a$ and $b$ and the product of $b$ and $c$ is even.
%% (Scores {tree_length = 70, tree_depth = 15, characters = 226, tokens = 72, subsequent_dollars = 0, initial_dollars =
0, parses = 3},386)

Prop110. Let $a$ and $c$ be instances of integers. Assume that we can prove that $a$ is odd and $c$ is odd. Then we can
prove that for all integers $b$, the sum of the product of $a$ and $b$ and the product of $b$ and $c$ is even.
%% (Scores {tree_length = 71, tree_depth = 14, characters = 230, tokens = 72, subsequent_dollars = 0, initial_dollars =
0, parses = 3},390)
```

```
module Informath2Core where

import Informath

data SEnv = SEnv {varlist :: [String]}

initSEnv = SEnv {varlist = []}

newVar :: SEnv -> (GIdent, SEnv)

sem :: SEnv -> Tree a -> Tree a
sem env t = case t of

  GAdjProp (GAndAdj (GListAdj adjs)) x ->
    let sx = sem env x
    in GAndProp (GListProp [GAdjProp adj sx | adj <- adjs])

  GAdjProp adj (GEveryKindExp kind) ->
    let (x, env') = newVar env
    in sem env'
      (GAllProp (GListArgKind [GIdentsArgKind kind (GListIdent [x])])
        (GAdjProp adj (GTermExp (GTIdent x))))
```

From Informath to Core is simpler:

- deterministic conversion of Informath extensions to MathCore
- like logical semantics (since MathCore is an unambiguous syntax for logic)
- fresh variables must be created for varless in situ quantifiers

Order is important:

every number is even or odd

→ *for all numbers x, x is (even or odd)*
→ *for all numbers x, (x is even or x is odd)*

→ ~~*every number is even or every number is odd*~~
→ ~~*(for all numbers x, x is even) or (for all numbers x, x is odd)*~~

Demos

make

```
all: Informath.pgf Dedukti Agda Coq Lean RunInformath

Informath.pgf:
	cd grammars ; gf --make -output-format=haskell -haskell=lexical --haskell=gadt
-lexical=Name,Noun,Fam,Adj,Rel,Fun,Label,Const,Oper,Compar,Set,Coercion,Relverb,Relno
un,Reladj,Comparnoun,Verb --probs=Informath.probs InformathEng.gf InformathFre.gf
InformathSwe.gf

Dedukti:
	cd typetheory ; bnfc -m -p Dedukti --haskell-gadt Dedukti.bnf ; make
Agda:
	cd typetheory ; bnfc -m -p Agda --haskell-gadt Agda.bnf ; make
Lean:
	cd typetheory ; bnfc -m -p Lean --haskell-gadt Lean.bnf ; make
Coq:
	cd typetheory ; bnfc -m -p Coq --haskell-gadt Coq.bnf ; make

RunInformath:
	ghc -package gf RunInformath.hs
```

```
demo:
	./RunInformath -lang=Fre test/exx.dk
	./RunInformath -lang=Swe test/exx.dk
	./RunInformath -lang=Eng test/exx.dk
	./RunInformath -lang=Eng test/exx.dk >exx.txt
	./RunInformath -lang=Eng exx.txt
	./RunInformath -lang=Eng test/gflean-data.txt

	cat BaseConstants.dk test/exx.dk >bexx.dk
	dk check bexx.dk

	./RunInformath -to-agda test/exx.dk >exx.agda
	agda --prop exx.agda

	./RunInformath -to-coq test/exx.dk >exx.v
	cat BaseConstants.v exx.v >bexx.v

	coqc bexx.v
	./RunInformath -to-lean test/exx.dk >exx.lean

	cat BaseConstants.lean exx.lean >bexx.lean
	lean bexx.lean

	./RunInformath -to-latex-file -variations test/top100.dk >out/top100.tex
	echo "consider pdflatex out/top100.tex"

	./RunInformath -to-latex-file -variations test/sets.dk >out/sets.tex
	echo "consider pdflatex out/sets.tex"
```

make demo

Generating synthetic data

For those who are interested just in the generation of synthetic data, the following commands (after building Informath with `make`) can do it: assuming that you have a `.dk` file available, build a `.jsonl` file with all conversions of each Dedukti judgement:

```
$ ./RunInformath -parallel <file>.dk > <file>.jsonl
```

After that, select the desired formal and informal languages to generate a new `.jsonl` data with just those pairs:

```
$ python3 test/jsonltest.py <file.jsonl> <formal> <informal>
```

The currently available values of `<formal>` and `<informal>` are the keys in `<file>.jsonl` - for example, `agda` and `InformathEng`, respectively.

https://github.com/aarneranta/informath/

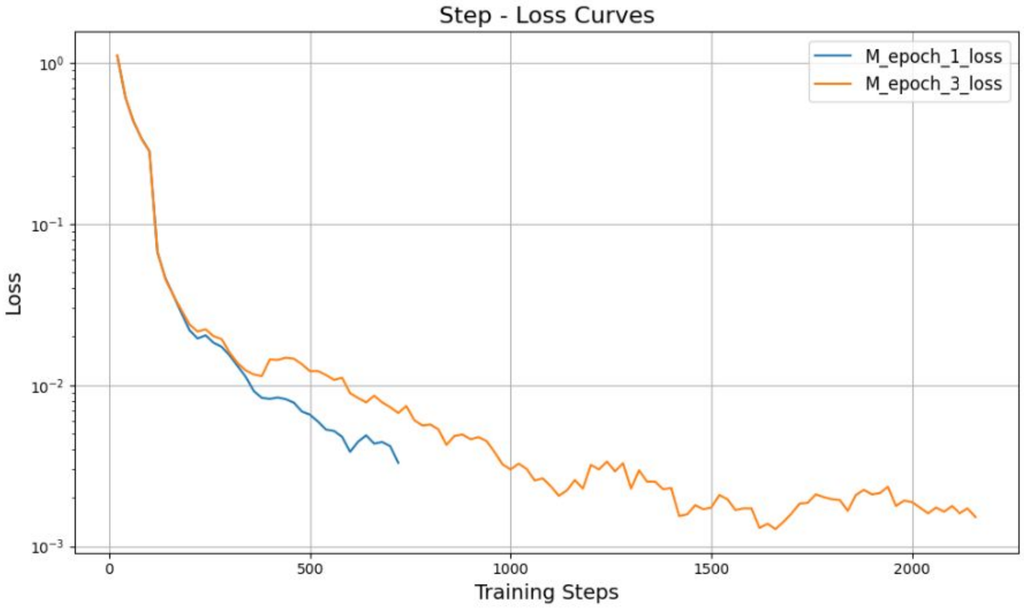


Figure 4.5: Training losses of fine-tuned models at different epochs.

Table 4.5: Model performance at different training epochs

Model	BLEU-4	ROUGE-1/2/L	Syntax Err.%	Score
Baseline	32.90	54.17 / 21.99 / 42.76	98.43	23.96
M_epoch_1	76.16	89.03 / 74.94 / 83.22	7.93	83.60
M_epoch_3	77.78	89.86 / 76.63 / 84.37	20.48	80.14

Fine-tuning an LLM:

- Qwen2.5-7B-instruct

Trained with ~1000 synthetic pairs of (dedukti,agda,coq,lean) - (English, French, Swedish) with

- arithmetic
- naive set theory
- concepts for 27 of the "100 theorems"

Tested with 57 natural native-speaker expressions of those theorems (by Nick Smallbone)

Pei Huang, *Autoformalization for Agda via Fine-tuning Large Language Models,* MSc thesis at Chalmers, 2025

Translate the following latex InformathEng translation problems (each is on one line) to deducti.
Do it analogously to the following extraction examples. Put the output all into one piece of text, preceded always by input (as in the examples).
BEGIN EXAMPLES
{"InformathEng": "thm58 . let $ A $ be a set . let $ n \\in N $. then $ | A | = n $, only if for all natural numbers $ k $, if $ k \\leq n $, then $ | \\binom{ A }{ k } | = \\binom{ n }{ k } $.", "dedukti": "thm58 : (A : Set) -> (n : Elem Nat) -> Proof (Eq (cardinality A) n) -> (k : Elem Nat) -> Proof (Leq k n) -> Proof (Eq (cardinality (combinationsFromSet A k)) (binomial n k)) ."}
{— …
END EXAMPLES
BEGIN TRANSLATION PROBLEMS
{"InformathEng": "Thm01f . there exists no natural number $ p $, such that there exists a natural number $ q $, such that the square of $ p $ is equal to the product of $ 2 $ and the square of $ q $.", "dedukti":
{"InformathEng": "thm52 . let $ A $ be a set . then we can prove that $ A $ is finite , only if we can prove that the cardinality of the power set of $ A $ is equal to the exponentiation of $ 2 $ and the cardinality of $ A $.", "dedukti":
— …
END TRANSLATION PROBLEMS

Sure, here are the translations of the given InformathEng statements to Dedukti:

```
input: {"InformathEng": "Thm01f . there exists no natural number $ p $ , such that
there exists a natural number $ q $ , such that the square of $ p $ is equal to the
product of $ 2 $ and the square of $ q $ .", "dedukti": "Thm01f : Proof ( not ( exists
Nat ( p => exists Nat ( q => Eq ( square p ) ( times 2 ( square q ) ) ) ) ) ) ."}
— …
```

https://chat.mistral.ai/chat/60f44ab2-d86f-4ec2-b382-d94fd471eef0 by Josef Urban, 13 July 2025

Conclusion

Symbolic informalization can be

- natural and fluent
    - by extending CNL towards the full language of mathematics
- feasible to develop
    - by Dedukti, GF, and rule extraction
- shared by different formal and informal languages
    - by Dedukti and GF interlinguas
- inverted to autoformalization
    - natively, by reversilibility of GF
    - as backup, by fine-tuned LLM + feedback informalization

Symbolic informalization is

- based on well-understood compiler-like techniques
- potentially 100% reliable
- fast and energy-efficient
- a natural extension of formal proof techniques

Building on the CNL tradition, the new things in Informath are

- wider coverage of alternative verbalizations
- multilinguality
- guaranteed grammaticality via GF RGL
- syntactic ambiguity allowed, resolved semantically

Some future work

Build up a multilingual lexicon with terms and definitions

- from Wikidata
- from Agda, Lean, Rocq, HOL-light, Isabelle, Mizar, …

Show competitive results in autoformalization

- learn from definitions, test with theorem statements

Improve the verbalization of proofs

- combine proof terms with scripts to identify crucial steps

Create APIs to connect with interactive proof systems

- use as a library or a plugin component

Don't guess if you know.

https://github.com/aarneranta/informath